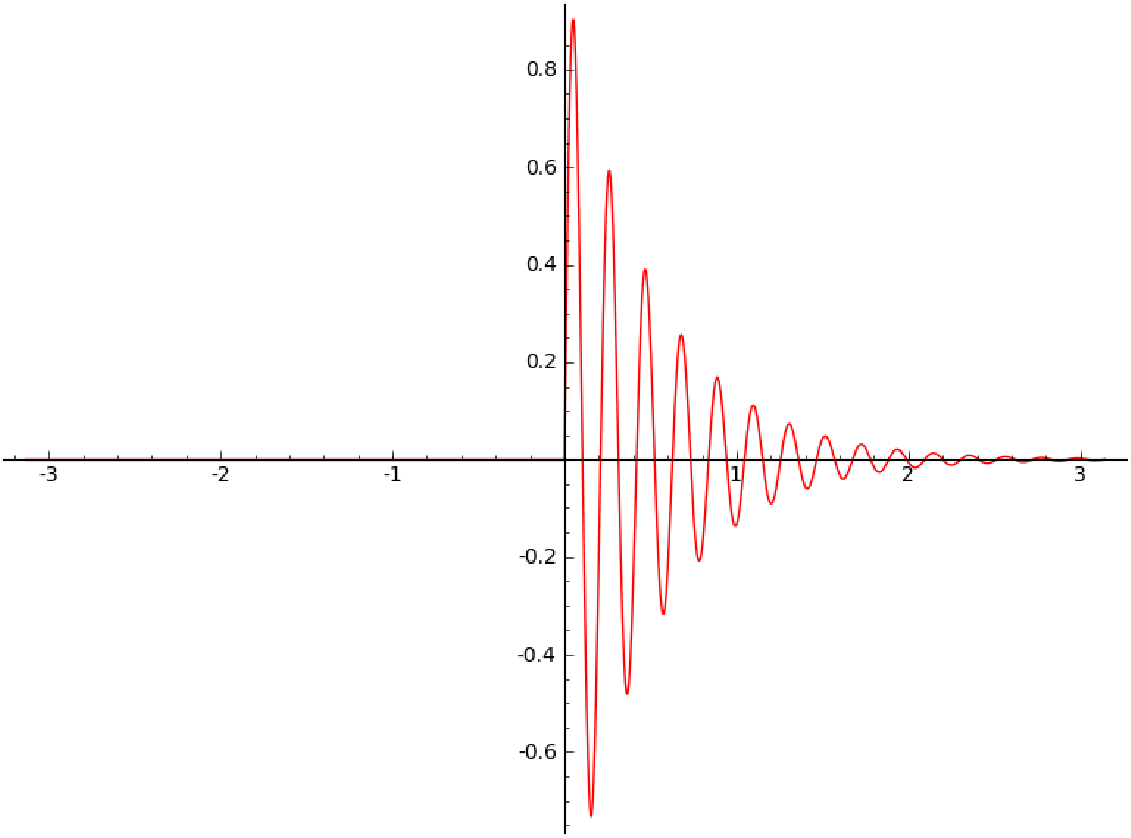
Fourier series in action: modeling a single-sided transient

Suppose we have a 'target function' in the form of a transient signal:
it *vanishes* for $t < 0$, but does something cool for $t > 0$.

Our example is motivated by the ringdown transient in an LCR system, or equivalent:
$f(t) = 0$ for $t < 0$
but
$f(t) = \exp(-2t) \sin(30t)$ for $t > 0$

We've chosen things so by the time we get to $t = +\pi$, the exponential is 'nearly dead'.



That function is neither even nor odd, so it needs *both* sine- and cosine-terms in a Fourier series.
Each Fourier coefficient can be computed analytically(!).
Clearly the n=30 sine term will have the biggest single coefficient, due to the $\sin(30t)$ content in the target function $f(t)$.

Here is the sum of

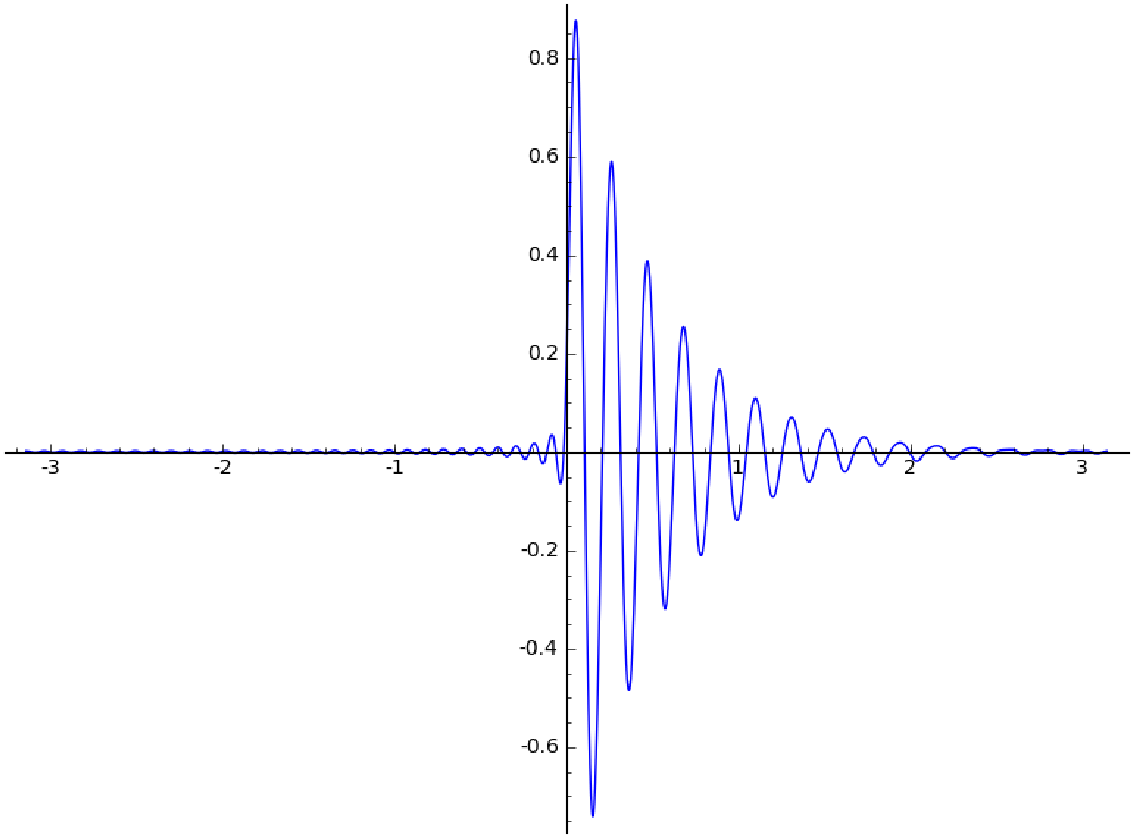
the dc term, $a_0/2$

plus

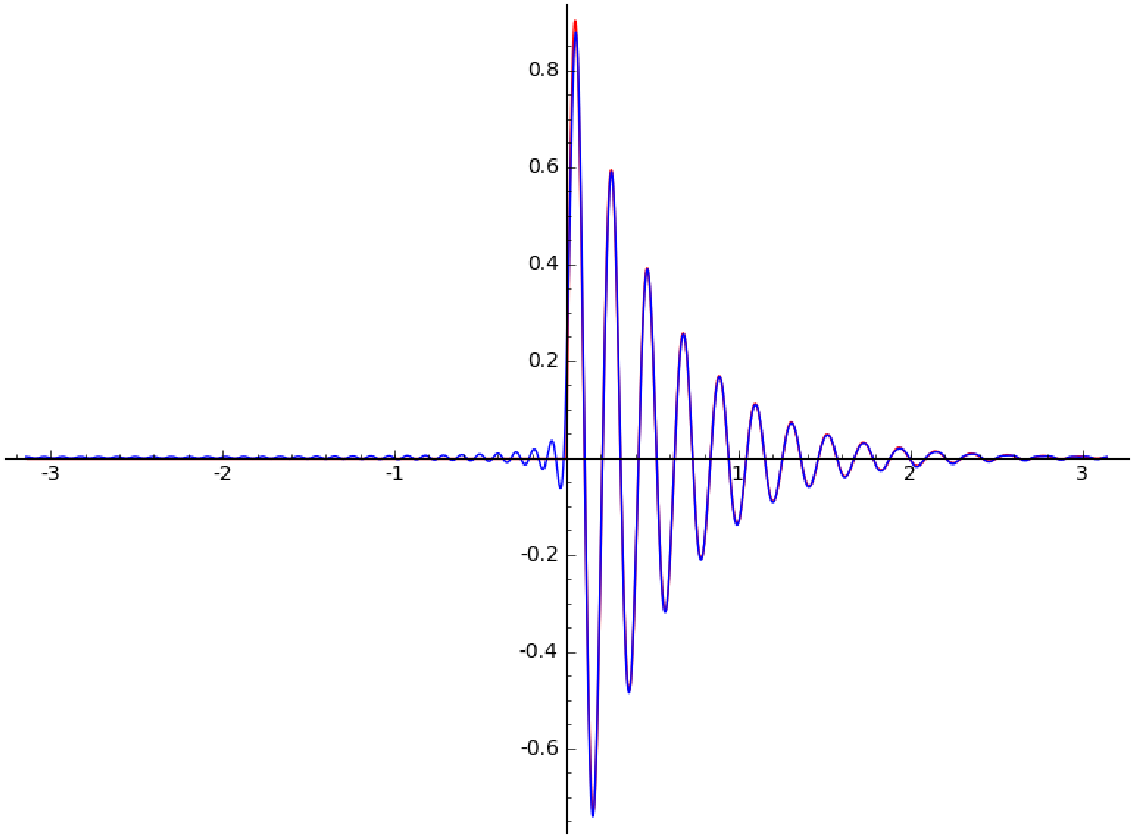
the cosine terms $a_n \cdot \cos(nt)$ for n=1 through n=59

plus

the sine terms $b_n \cdot \sin(nt)$ for n=1 through n=59:

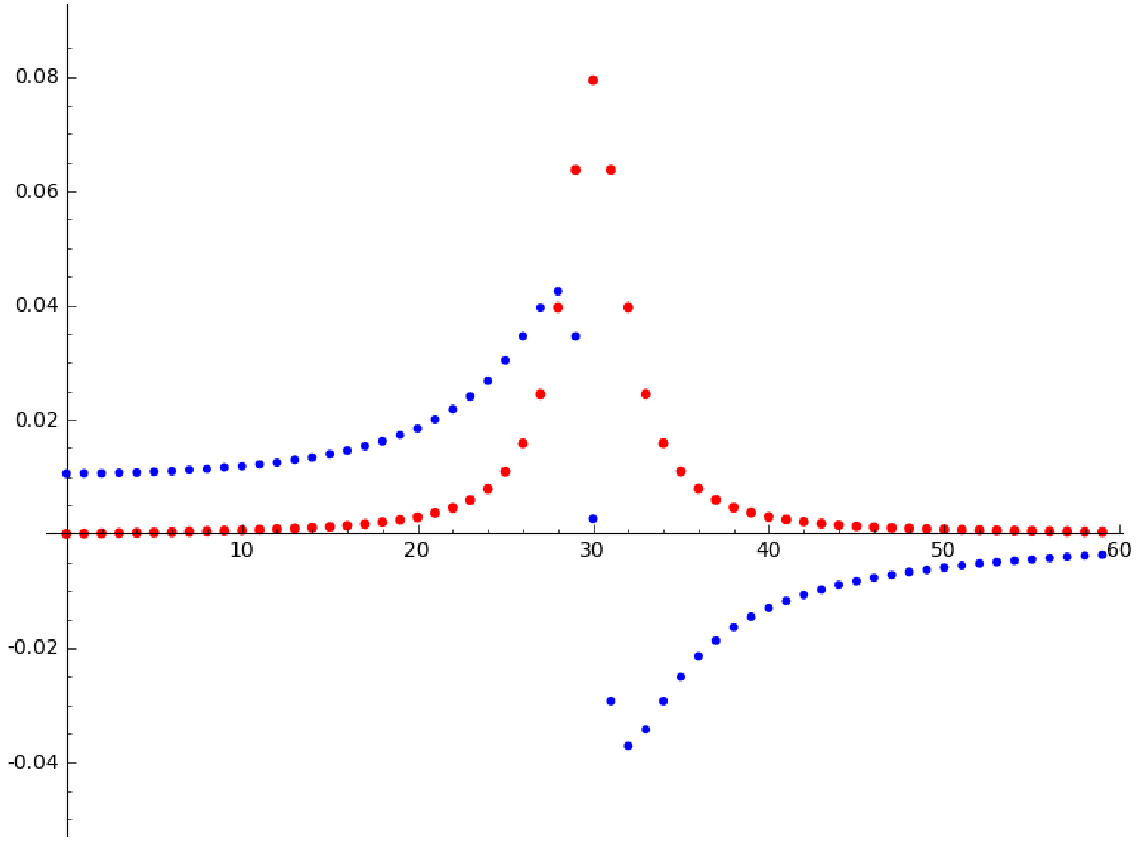


Here’s the overlay of the Fourier series in blue, and the target function in red:



The difference is pretty small! and contains only sinusoids of argument (60*t*) and higher.

Here is a plot showing the values of the coefficients a_n and b_n as a function of n:
Cosine coefficients are in blue, sine coefficients in red (and bigger) dots:



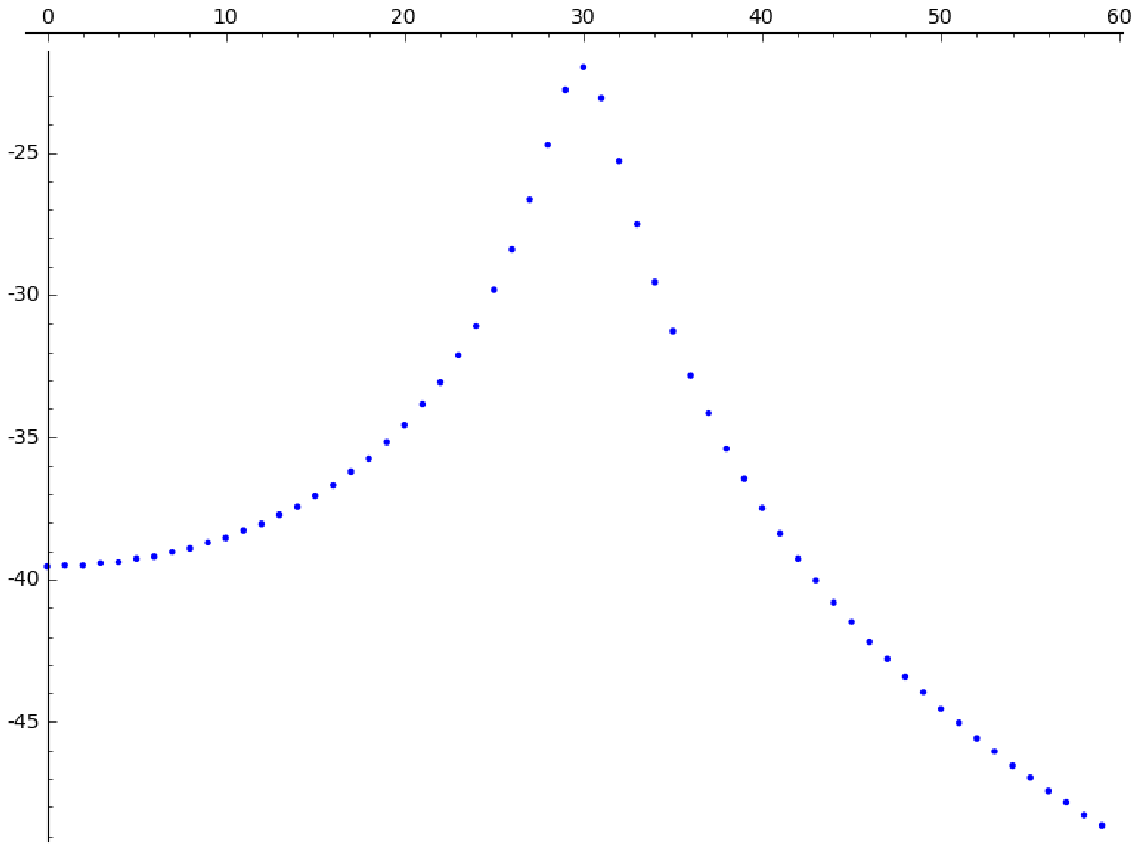
Notice the biggest single coefficient is red, ie. a sine-coeffcient, and it does occur at n=30 as expected.

Notice the sine-coefficients fall along an ‘absorption profile’ and (necessarily) contain content other than just n=30;
Notice the cosine coefficients fall along a ‘dispersion curve’.

The peak and valley of the dispersion curve are located, along the n-axis,
right at the half-maximum points of the absorption curve.

Next, the *magnitudes* are the root-sum-square of sine and cosine coefficients.

Here I plot 20*$\log_{10}$(of each magnitude),
to create the effect of a dB scale:



and that shows the Lorentzian profile we'd expect, with peak near *n*=30.