ALPhA Immersions, July 2013, 'Fourier Methods'
'Homework' for participants

You should have read a 'Chapter 0' .pdf for a conceptual overview of Fourier Methods, and a 1-page .doc excerpt called Immersions2013-Definition, before trying this homework. How you solve these problems below will depend on what analytical, computational, and numerical resources you like to use. You are free to submit responses (or questions) to me in a .doc, or a scanned handwritten effort, by e-mail to dvanbaak@calvin.edu or via FAX to (716) 836-1077. No need for publication-quality typesetting or graphics!

1. The 'Definitions' document defines a Fourier series whose basis is a set of *complex* exponentials in time, multiplied by *complex* coefficients b_k. Show that the Fourier series expansion for a real-valued periodic function has property $b_k = (b_{-k})^*$ where * means complex conjugation. Show that this gives (for example) (Re b_{-1}) = (Re b_1) and (Im $_{b\text{-}1}$) = -(Im b_1).
[That, in turn tells you that knowing b_1 fully determines b_{-1}, or in general that you only need to compute the b_k for $k \geq 0$ to know them for all k.]
[Series in complex exponentials have immense advantages in numerical use of the famous Cooley-Tukey FFT algorithm, which motivates this choice of 'basis functions'.]

2. Given a real-valued function $V(t)$ which is *even* [so $V(-t) = (+1)V(t)$], show that all the b_k-coefficients are real as well. Use this to show that the series expansion of $V(t)$ in complex exponentials reduces to an expansion in cosine functions only.
[That is to say, such a $V(t)$ has an expansion in a dc-term plus cosines, but no sine-content at all.]

3. The function $V(t) = 4 / [\ 5 – 3 \cos (2\pi\ t / T)\ ]$ has period T, and is real-valued and even. Compute its dc-average b_0, and the Fourier coefficients b_1 and b_2 as well. Numerical methods are fine; and you are free to choose the period $T = 2\pi$ for convenience. Now plot the $V(t)$ function over the interval $–2 < t < 8$, and overlay that plot with the 5-term Fourier series involving the terms k = -2 through k = +2 inclusive.
[The point is to show how well this function can be approximated by a constant plus only two cosines. If you like to go on to *four* cosines, you might be more impressed.]

4. Suppose $V(t)$ is periodic with period T, and that its $0 < t < T/2$ behavior gets 'shifted and flipped' into the $T/2 < t < T$ interval such that $V(t + T/2) = (-1)\ V(t)$ for any t.
Show that the coefficients b_k *vanish* for all even k-values.
[This entails that its dc-average b_0 vanishes, and that all its even harmonics do too, so the 'Fourier power' must all be contained in the k = 1 fundamental and *odd* harmonics.]

PS. This is the <u>last</u> analytical/computational work you'll need to do in this Immersion, and everything in our time together will be hands-on, electronic, and 'lab-like'. But this will help you with some of the conceptual vocabulary of our Immersion.