The Index of Refraction using the Pfundt Method

An intermediate level lab with a conceptual component

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1. Introduction

The intermediate or sophomore-level laboratory for the physics major may vary considerably among institutions. Its objectives and content are influenced by the nature of the introductory or freshman laboratory (traditional, conceptual or a mixture thereof) as well as what courses are normally taken concurrently by the sophomore level student. There may also be varying goals and objectives for the intermediate laboratory experience.

At Loyola University Maryland we do not require an optics course, so this and other labs in geometrical and physical optics are an important component of our intermediate lab curriculum. This lab exercise is usually done in fall of the sophomore semester, that introduces a method to measure the index of refraction of transparent solids and liquids. It serves as a relatively simple lab requiring no more than two lab periods. Not only does this exercise introduce students to a phenomenon in geometric optics they most likely have not seen previously, it provides a simple exercise in ray tracing, it requires straight-forward but non-trivial error analysis, and it provides a conceptual link to theory behind fiber optics phenomena.

Depending on the emphasis of the instructor, this lab can have multiple benefits:

- This can be used as a conceptual exercise in applying concepts of geometric optics. Based simply on their observations and their understanding of Snell's Law, students are asked to explain the phenomena they observe.
- The error analysis is neither trivial nor overly complicated. It can provide a refresher of what was introduced in the freshman laboratory course or serve as an example of more sophisticated error analysis being introduced at the sophomore level.
- Minimal equipment is needed. The set-up of the equipment is easy and not time-consuming. No equipment problems can interfere with the purpose of the lab.
- Students can be creative in developing a technique to measure the ring diameters. The value of multiple observations is shown. With care uncertainties of 2% or less can be obtained.

Different lab instructors at Loyola use different approaches. The Student Lab Handout included here is a recent variation. Besides this Lab Handout included are a set of Instructor Notes that detail various techniques for measuring the ring diameters and show typical patterns observed, as well as measurements and results. Included also is a Conceptual Exercise which can be used as an alternative approach.

2. Equipment List

The equipment needed is simple, can usually be found in the laboratory or can be purchase for under $50. If the instructor sees value in using a slab of glass of known refractive index for students to compare with their results, such are available but will increase the cost.

- Laser source (preferably green to reduce eye strain; red works fine)
  Green laser pointer such as available from Arbor Scientific #P2-7677 $49
  Other lasers work fine.
- Glass and/or acrylic plate. We've used an optical flat, Edmunds # 32644, a 4 inch diameter disk of fused quartz ¾ inch thick with n = 1.458. However, any flat plate with thickness over about 0.5 cm works well.We've used assorted acrylic disks, even the glass shelf from a bathroom cabinet
  - The bottom should be painted with flat white paint to provide a reflecting surface for the rays striking the bottom of the plate. One small section part must be left unpainted in order to measure the thickness of the plate.
  - It is instructive to leave about half the bottom surface unpainted so rays reflected from both the painted and non-painted bottom surface can be compared simultaneously.
  - Since acrylic may be dissolved by the paint's solvent we use gesso on the acrylic plate.
- Miscellaneous items;
  Ring stand and clamps, condenser clamp, ruler, Vernier or micrometer caliper, white paper sheet, millimeter graph paper, several ml of water or other liquid.

Also included are

PH 293, 294: Intermediate Laboratory 2017

Index of Refraction - STUDENT

Introduction

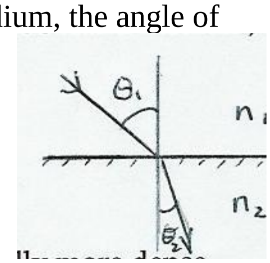
When a wave (or particle) travels from one medium to another in which it moves with a different velocity, the media can be characterized by the relative index of refraction:

$$\text{Relative index of refraction} = \frac{n_2}{n_1} = \frac{v_1}{v_2}$$

For light waves whose velocity in a vacuum is traditionally designated as c, we can also define the refractive index of a medium by comparing it to a vacuum ($n_{vacuum} = 1$) as

$$n = \frac{c}{v} \quad \text{where c is the speed of light in a vacuum}$$

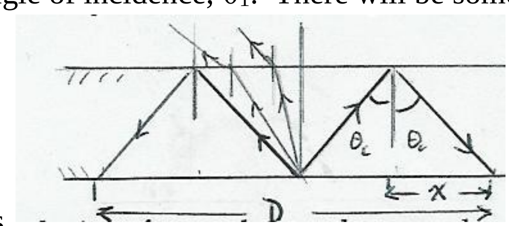
If a light ray (wave or particle) is incident at some angle θ on an interface between two different media, then the change in velocity will cause the direction of the ray to also change. August Snell, treating light rays as having wave properties, developed a relation between the angle of the incident ray on the interface, θ_1, and the angle of its path in the second medium, the angle of refraction, θ_2. Specifically what we now call Snell's Laws states:



$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} \qquad \text{or} \qquad n_1 \sin\theta_1 = n_2 \sin\theta_2$$

An interesting effect can occur when a light ray travels from an optically more dense medium to one that is less dens ($v_1 > v_2$) where the angle of refraction, θ_2, will always be greater than the angle of incidence, θ_1. There will be some critical angle θ_c where θ_2 will equal 90° such that

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{\sin 90°}{\sin\theta_c} = \frac{1}{\sin\theta_c}$$



In this case the ray effectively does not pass through the interface and enter the second medium but is reflected back into the incident medium at the interface. This effect is called total internal reflection. Total internal reflection provides a quick and surprisingly accurate method of measuring the index of refraction of transparent media such as glass, and of liquids.

If a point source is located at the bottom surface of a piece of plane parallel glass, it will send rays upward in all directions. Depending on the angle of incidence at the upper surface, the interface, many will pass through. But at a certain critical angle the rays are reflected back into the medium and if the underside of the glass is coated with white paint these critically reflected rays form a ring of light that has a quite well-defined edge which allows a precise measurement of the ring diameter D. The ring diameter can be related to the sine of the critical angle from the geometry of this arrangement,

$$\sin\,\theta_c = \frac{x}{\sqrt{x^2+t^2}} = \frac{D}{\sqrt{D^2+16t^2}}$$ where t is the thickness of the glass plate

so

eq 1) $$n_g = \frac{\sqrt{D^2+16t^2}}{D}$$

If a liquid is placed on the surface of the glass plate, a second ring of light can be seen where

$$\frac{n_2}{n_1} = \sin\vartheta_c = \frac{x}{\sqrt{x^2+16t^2}}$$

or

eq 2) $$n_{liquid} = n_{glass}\frac{x}{\sqrt{x^2+16t^2}}$$

This effect is the basis of how many commercial refractometers are constructed.

Prelab

1. Derive Eq. 1 for a piece of glass of thickness t. The glass has a bottom surface covered in white paint. Air is above the top surface.
2. Draw a more complete ray tracing diagram than the one shown above. The incoming beam heads straight down. There is a diffuse reflection off the white paint at the bottom of the glass. Draw the path of at least 11 rays.

Experiment

Part 1

Two quantities need to be measured in order to calculate the index of refraction of the given piece of glass: the thickness *t* of the glass and the diameter *D* of the ring of light. The object of this lab is to make the most precise determination of the index that you can and by using the appropriate uncertainties in these measurements, calculate the uncertainty in this index of refraction.

Use a micrometer to measure the thickness of the glass. Each person should measure the thickness in a variety locations around the edge of the glass, and by combining measurements compute an average and standard deviation for your piece of glass.

Record the color and the wavelength of the laser beam.

Take many measurements of the ring diameter and quote an average value and uncertainty. (What do you think is reasonable to use as the uncertainty: the precision of the measuring instrument or the standard deviation of the measurements?)

Calculate the index of refraction for a given wavelength.

Develop an equation in symbols that relates the uncertainty in the calculated index Δn to the uncertainties in your measured quantities, Δt and ΔD. Implement your calculation in Mathematica, where values are set at the top of the notebook, and the calculation is conducted below. The end result of the Mathematica calculation is the absolute uncertainty in the index of refraction.

Compare the measured range of values, expressed as $x_{best} \pm \delta x$ with the accepted value. Read Taylor section 2.4, pp. 18-20, to understand how to write about the agreement between the measured values and the accepted value. Calculate the percent error between your experimental value and the accepted value.

Part 2

Previous results have shown a discrepancy between the measured value for the index of refraction and the accepted value. Try to understand more carefully what the discrepancy could be due to. Situations to consider are:

a. Is the edge of the circle sharp or blurred?
b. What would be the effect of placing a piece of white paper on top of the glass?
c. Would the result for n be dependent on the wavelength of light? In what way?

Redo the measurements described in Part 1, but this time place a piece of damp, white paper on top of the glass. Calculate the index of refraction and the uncertainty. Compare the measured range of values with the accepted value. Discuss your results in Parts 1 and 2. Which result was more accurate and why?

In the report, the derivation and ray tracing should be inserted in the theory section, which is positioned right after the Introduction. Include the measurements and analyses for Parts 1 and 2. Photograph the phenomenon and label what you measured. Each person should submit his/her own Mathematica notebook on Moodle. The best notebook should be attached to the report in the Appendix.

Other activities to try

1. The writeup describes what happens when a liquid is placed above the glass plate. Try water. Photograph the phenomenon, and use a ray tracing to understand the results. Also derive Eq. 2.

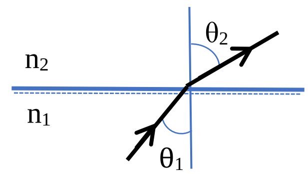
2. Repeat the measurement for a different wavelength. Compare and discuss your results for both wavelengths.

4. Instructors Notes

A. Developing a working equation

Snell's Law states $n_1 \sin\theta_1 = n_2 \sin\theta_2$

so when $n_1 > n_2$ then $\theta_2 > \theta_1$

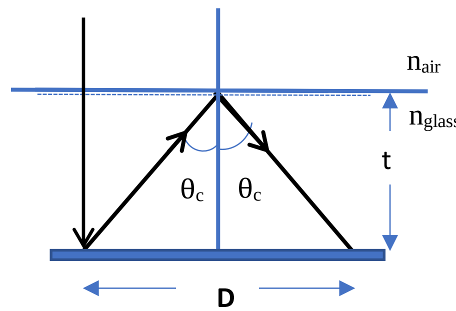


Index of refraction of glass or acrylic

At a glass to air interface, $n_{glass} \sin\theta_{glass} = n_{air} \sin\theta_{air}$ and $\theta_{air} > \theta_{glass}$

When $\sin\theta_{air} = 90°$ and letting $n_{air} = 1$

$$n_{glass} = \frac{1}{\sin\theta_c}$$



From the geometry of the ray diagram,
where t is the plate thickness and D the diameter of the ring.

$$\sin\theta_c = \frac{D/4}{\sqrt{(D/4)^2 + t^2}} = \frac{1}{\sqrt{1+(4t/D)^2}}$$

$$n_{glass} = \sqrt{1 + \left(\frac{4t}{D}\right)^2}$$

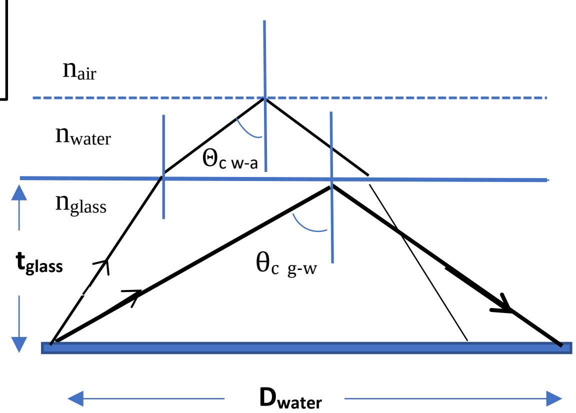
Index of refraction of water or a liquid

If a layer of water is placed on top of the glass, Snell's Law at the **glass-water interface** becomes

$$n_{glass} \sin\theta_{glass} = n_{water} \sin\theta_{water}$$

so when a ray is incident at the critical angle, $\sin\theta_{water} = 90°$

$$n_{water} = n_{glass} \sin\theta_{c\ glass\text{-}water} = n_{glass} \frac{1}{\sqrt{1+\left(4t_{glass}/D_{water}\right)^2}}$$



Error Analysis

Using the rules that uncertainties are added when adding or subtracting quantities,
And relative errors are added when multiplying or dividing.

$$n_g = \sqrt{1+\left(\frac{4t}{D}\right)^2}$$

We can make some useful substitutions,

let $Y = \left(\frac{4t}{D}\right)$ so then $\frac{\delta Y}{Y} = \frac{\delta t}{t} + \frac{\delta D}{D}$ $\qquad n_g = \sqrt{1+Y^2}$

and $X = Y^2 = \left(\frac{4t}{D}\right)^2$ so $\left(\frac{\delta X}{X}\right) = 2\left(\frac{\delta Y}{Y}\right) = 2\left[\frac{\delta t}{t} + \frac{\delta D}{D}\right]$ $\qquad n_g = \sqrt{1+Y}$

then let

$Z = 1 + \left(\frac{4t}{D}\right)^2$ $\qquad \delta Z = \delta X = 2\bullet X\left[\frac{\delta t}{t} + \frac{\delta D}{D}\right] = 2\left(\frac{4t}{D}\right)^2\left[\frac{\delta t}{t} + \frac{\delta D}{D}\right]$ $\qquad n_g = \sqrt{Z}$

and finally

so $\qquad \frac{n_g}{n_g} = \frac{1}{2}\left(\frac{\delta Z}{Z}\right) = \frac{\left(\frac{4t}{D}\right)^2\left[\frac{\delta t}{t} + \frac{\delta D}{D}\right]}{1+\left(\frac{4t}{D}\right)^2}$

$$\boxed{\delta n_g = \frac{(4t/D)^2}{1+(4t/D)^2}\left[\frac{\delta t}{t} + \frac{\delta D}{D}\right]}$$

B. Measuring the rings with different techniques

Where the plate it has not been painted, its plate thickness can be measured with a vernier or a micrometer caliper.

The ring diameter D can be tricky to measure since the edge is a bit fuzzy. This is where multiple measurements and using the average or standard deviation as the uncertainty is instructive

Suggestions for measuring the diameter are:

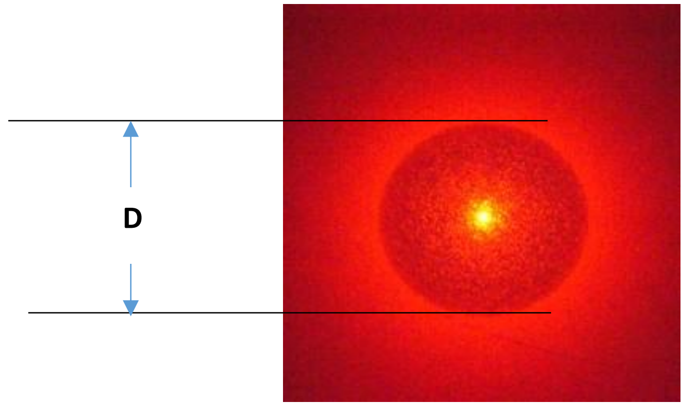
- Use a vernier caliper to measure either the inner or outer ring diameter. Since viewing the pattern on the bottom can introduce parallax, try to look downward from a point a the middle of the jaws opening. This will likely give the largest distribution of values.

- It may be easier to use the edge of the painted surface as it clearly shows a diameter. Repeated measurements by one or more students will still give a nice uncertainty (average or standard deviation) in the measurement.

- Place a millimeter-scaled graph paper under the plate. The graph paper can be easily adjusted so a line is coincident with the ring diameter viewed on the painted surface. This avoids the parallax problem but 'instrument' uncertainty is only 0.5 millimeter. Adjusting and taking multiple readings can increase the sensitivity of this method.

- Place a piece of damp, white paper on top of the glass. Use a caliper to directly measure the diameter of the ring seen at the top surface. Note this ring occurs at the point of critical refraction and is the half of the diameter of the ring seen on the bottom surface. Note how $\delta \mathrm{D}$ is related to δR.

- Replacing the painted bottom surface with a piece of photographic paper was evidently once a common technique that has fallen out of favor.



C. Typical data and results

Students are required to implement their calculations in Mathematica, where values are set at the top of the notebook, and the calculation is conducted below, or to set up and Excel spreadsheet as is done below

Optical Glass

Av. Plate Thickness t	Av. Ring Diameter d	4*t/d	sqrt [1+(4t/d)^2]	sinθ_c	n_{glass} = n_2/sinθ_c	n_{water} = n_{glas}*sinθ_c	**average n_x**	+/-
1.8833	6.8983	1.092037	1.48072	0.67535	1.48072			
1.8833	6.890	1.093353	1.48170	0.67490	1.48170			
18.825	68.983	1.091573	1.48038	0.67550	1.48038			
18.825	68.90	1.092888	1.48135	0.67506	1.48135			
				av	1.48104		**1.481**	0.008

Acrylic Disk

1.178	4.2167	1.117462	1.49957	0.66686	1.49957			0.010
1.178	4.1633	1.131794	1.51028	0.66213	1.51028			0.011
				av=	1.50493		**1.50**	.01
1.178	9.4617	0.498008	1.11714	0.89514		1.34718		
1.178	9.190	0.512731	1.12379	0.88985		1.33922		
					av=	1.34320	**1.343**	0.001

Rectangular Plate

1.194	4.272	1.117978	1.49996	0.66669	1.49996		**1.50**	0.038
Water								
1.194	8.872	0.538323	1.13569	0.88052		1.32519	**1.33**	0.038

D. A Conceptual Approach to the Pfundt Pattern

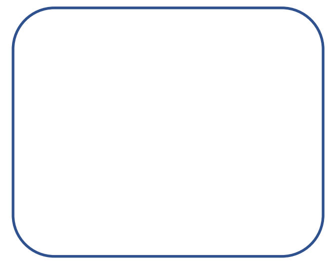
This lab exercise can also be approached as a conceptual exercise. Students can use their observations of dark and bright regions of the Pfundt patterns and Snell's Law to then sketch the paths of various rays to explain what they are observing.

Observations and Explanations

1. a. What happens when a laser beam is incident on a white paper surface?

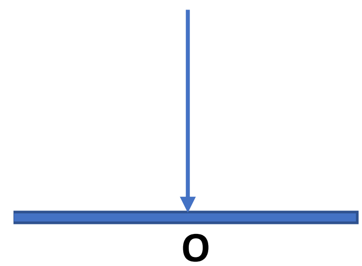
   Shine the laser beam directly on the white paper. Sketch and describe what you observe in terms of dark and bright regions.

Top View



   Sketch rays to explain what you observe.

Side View

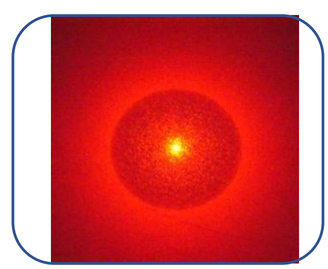


   b. Slide a glass or acrylic plate over the paper such that the beam now passes through it before being incident on the paper. Do you notice any change or variations in brightness? We'll return to this question after a few more observations.

2.

   a. Now place a plate which has had its underside painted white, thus becoming a good reflecting surface. Observe and sketch the pattern of bright and dark areas that you see.

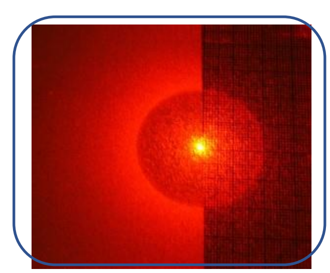
Top View



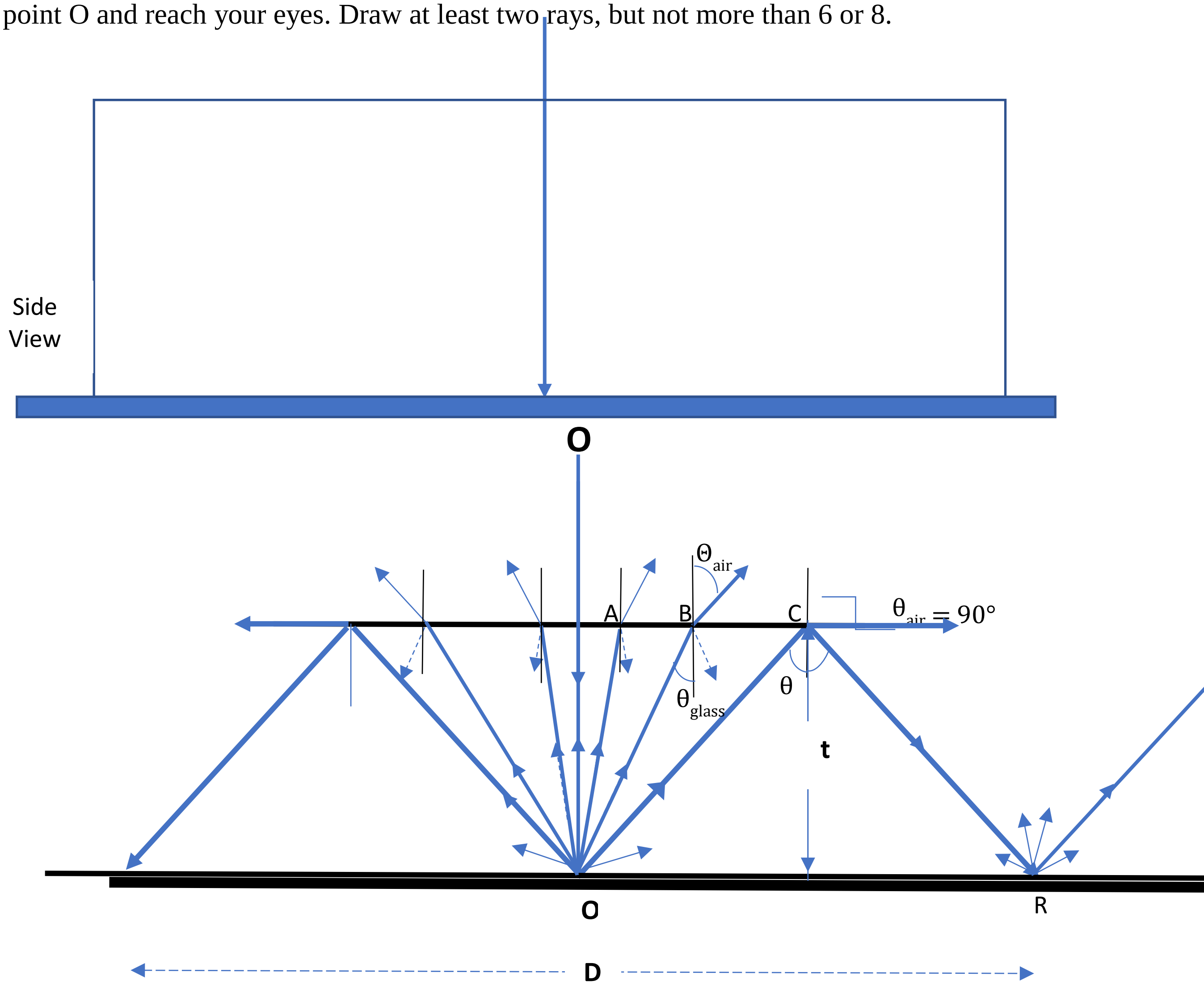
   b. You may want to also look at and sketch the pattern that results if you slide the plate such that the laser beam just touches the edge of the painted underside. Sketch or describe this pattern.

   Compare the patterns of the light and dark areas in the two halves of the disk.

Top View



c. Explain why you see this pattern by drawing a diagram of rays which come from point O and reach your eyes. Draw at least two rays, but not more than 6 or 8.



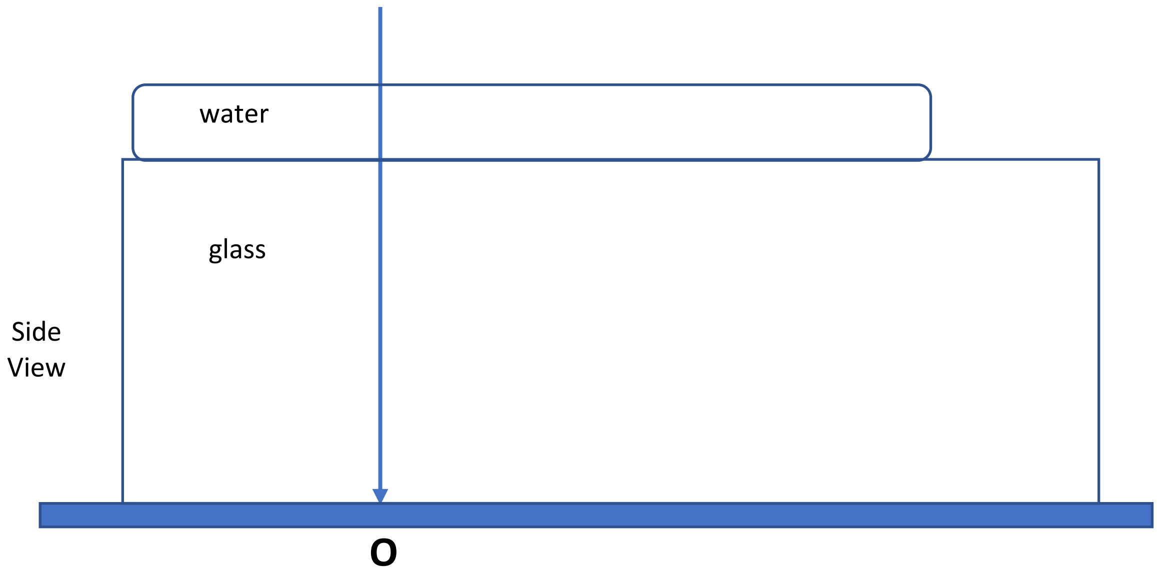
Rays between the incoming beam and point C such as rays A and B, come from the diffuse source O, and are refracted when they leave the upper plate surface. They are seen as forming a darkish disk, but one that is not completely void of light. While it looks 'black' in the first photo, this is an effect of the relative illumination; in the half-covered disk it is clear that you see some rays coming from that area.

At point C the ray is incident at the critical angle and it and rays close by are reflected back to the bottom of the glass, becoming visible as a bright ring. Point R then serves as a secondary diffuse source such that the pattern can be repeated faintly.

Note the pictures and ray diagram are include as notes for the instructors. They are not intended to be included in the student handout.

3.

Use the dropper to place some water over the top surface of the glass. Describe how the pattern changes and again draw rays to explain the pattern you see.



There will be one ray that comes from O, is refracted at the glass water interface, and reaches the top water surface at the critical angle. It will follow a path back to the bottom surface that 'mirrors' its outgoing path.
There will also be a ray that comes from O and strikes the glass-water surface at a critical angle and is reflected to the bottom surface.
Thus 2 rings are formed, the smaller from the water-air interface and the larger from the glass-water interface. Since the latter depends only on the thickness of the glass, which has been found, the equations similar to those derived for the glass-air interface can be used.