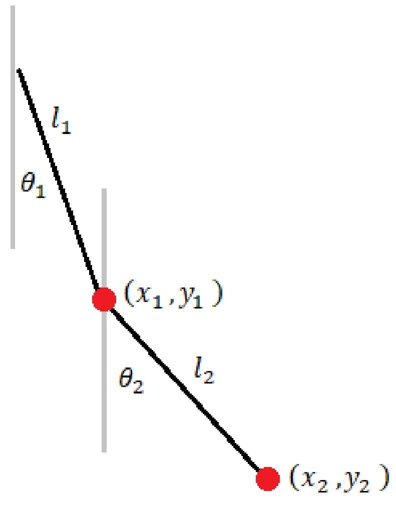
Double pendulum



Experimental conditions:

- Rigid connections of pendulum
- Good bracket
- Counterweights

Physical analysis by Lagrangian mechanics

(for ideal pendulum)

Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

L = kinetic energy – potential energy

Energies of double pendulum:

$$E_p = -(m_1 + m_2)l_1 \cos(\theta_1) - m_2 l_2 \cos(\theta_2)$$

$$E_k = \frac{m_1 {l_1}^2 \left(\frac{d\theta_1}{dt}\right)^2}{2} + \frac{m_2\left({l_1}^2\left(\frac{d\theta_1}{dt}\right)^2 + {l_2}^2\left(\frac{d\theta_2}{dt}\right)^2 + 2l_1 l_2 \frac{d\theta_1}{dt}\frac{d\theta_2}{dt}\cos(\theta_2 - \theta_1)\right)}{2}$$

Solution of Lagrange's equation which gives system of differential equations:

Forθ_1:

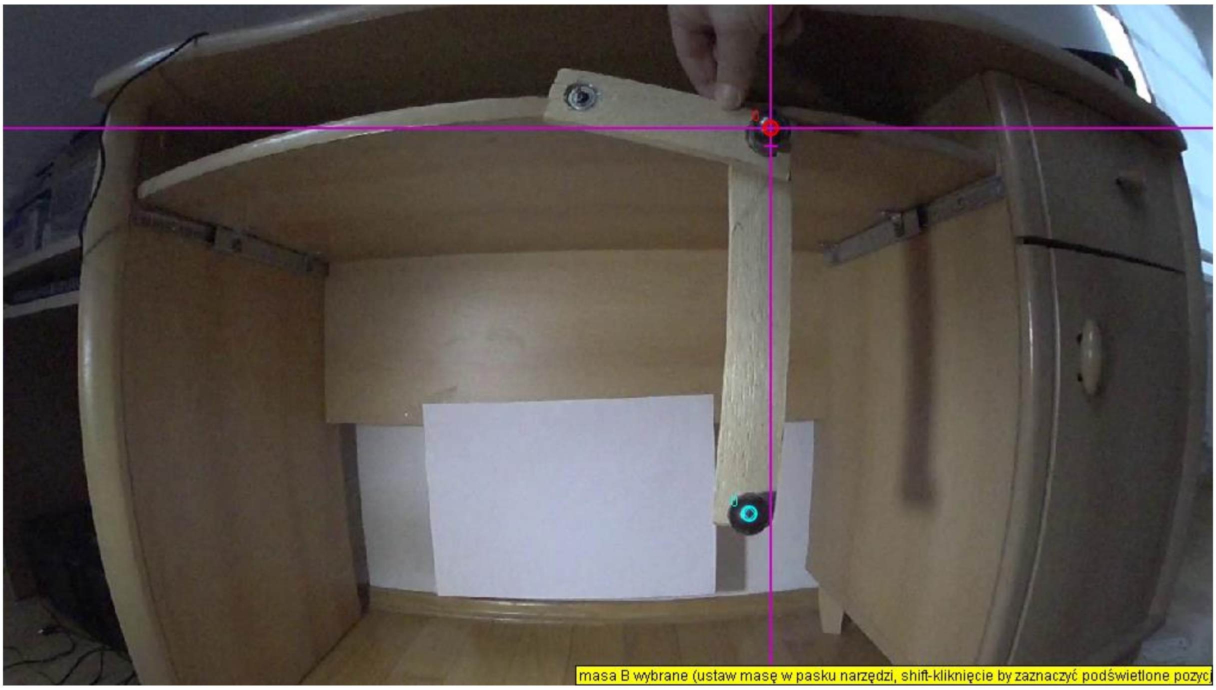
$$(m_1 + m_2)l_1\frac{d^2\theta_1}{dt^2} + m_2 l_2\frac{d^2\theta_2}{dt^2}\cos(\theta_2 - \theta_1) + m_2 l_2\frac{d\theta_2}{dt}\left(\frac{d\theta_2}{dt} - \frac{d\theta_1}{dt}\right)\sin(\theta_2 - \theta_1) + m_2 l_2\frac{d\theta_2}{dt}\frac{d\theta_1}{dt}\sin(\theta_2 - \theta_1) + g(m_1 + m_2)\sin(\theta_1) = 0$$

Forθ_2:

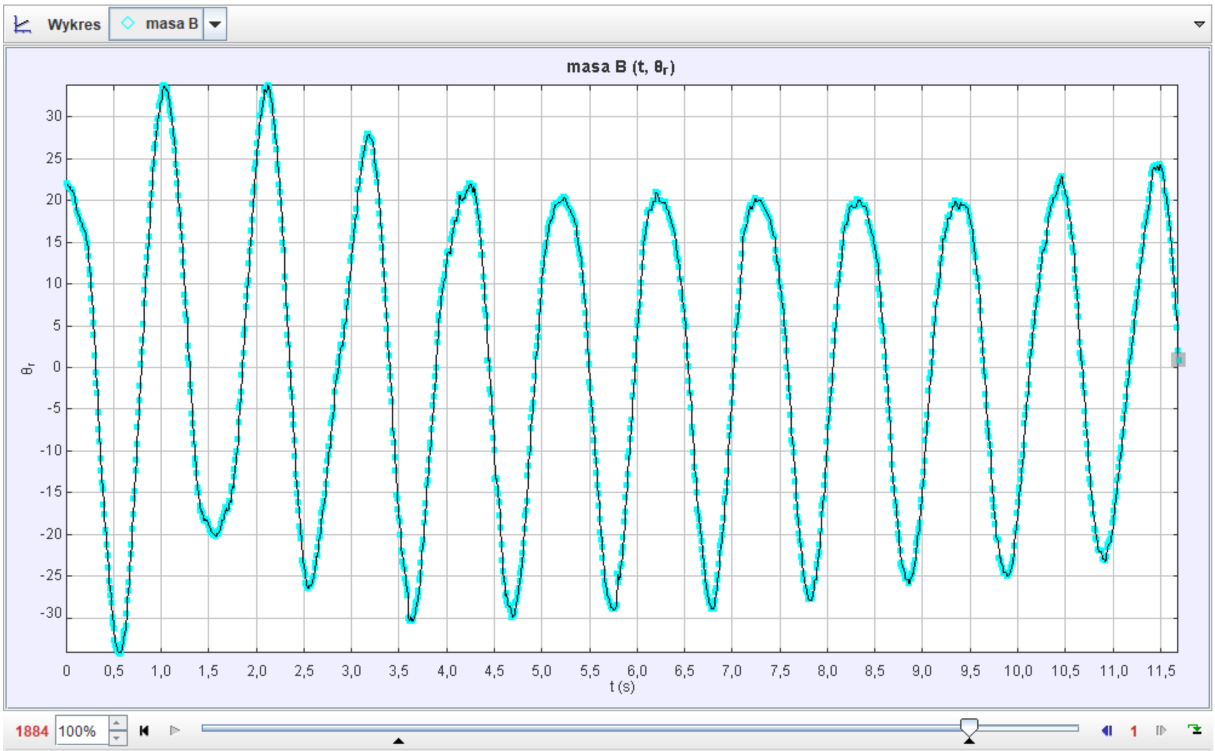
$$m_2 l_2\frac{d^2\theta_2}{dt^2} + m_2 l_1\frac{d^2\theta_1}{dt^2}cos(\theta_2 - \theta_1) + m_2 l_1\left(\frac{d\theta_1}{dt}\right)^2 sin(\theta_2 - \theta_1) + m_2 gsin(\theta_2)$$

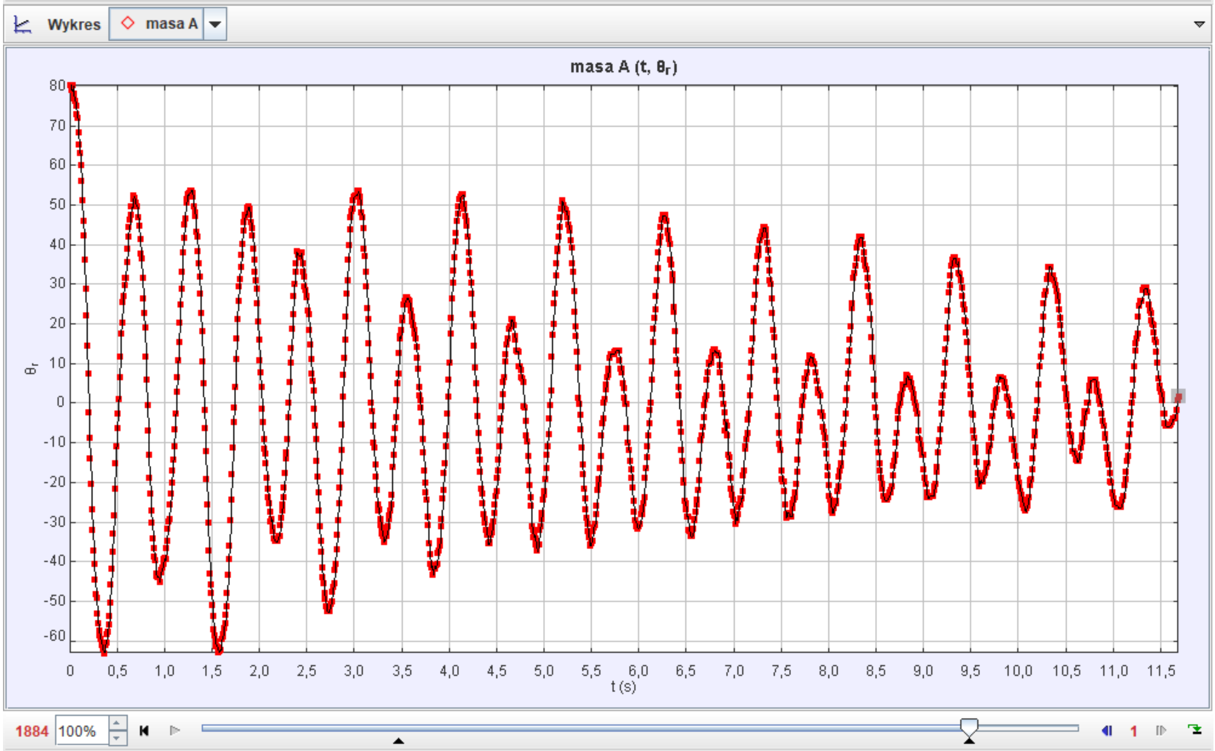
Numerical solution of this differential equation system gives us general function of angles (two degrees of freedom).The function highly depends on initial conditions.

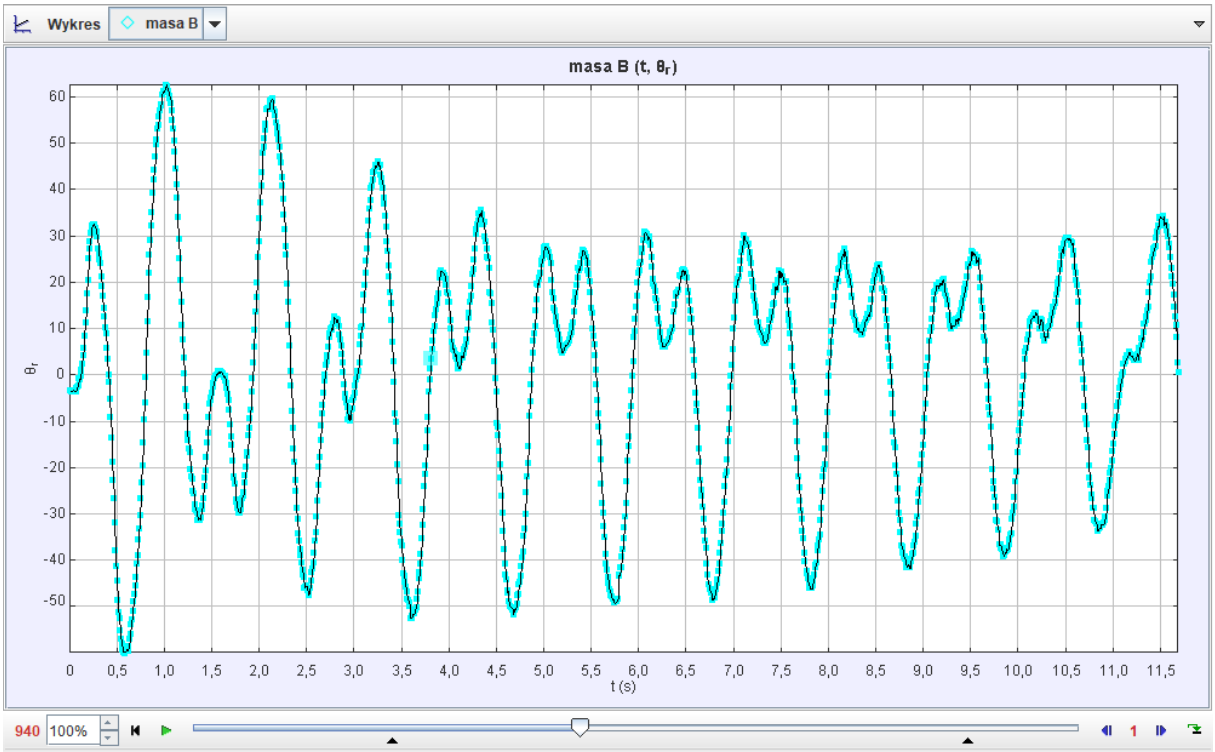
In this experiment we built a double pendulum, made of two wooden boards with different lengths and brackets. We leant out the middle bracket for a certain angle as shown on the photo. We put the construction into motion.



We were analyzing the movement of the middle and bottom points in time. We marked the middle point as Mass A and the bottom point as Mass B.







The first and the second plot illustrates changes in locus of Mass B and Mass A respectively, measured in regard of the point where the construction is pendant. It is easy to see that the amplitude of the oscillation is decreasing, because of the air resistance. We can also assume that the function of that change should be represented by some trigonometric functions. The oscillation of Mass A has higher frequency. Because of that Mass A and B are in opposite phases of their oscillations periodically.

The last plot illustrates the change in locus of Mass B in regard of the locus of Mass A. Here we can also assume that the amplitude is decreasing, because of the air resistance and that the function should be represented by some trigonometric functions. We can observe that periodically the movement of the pendulum is chaotic for some fluctuations. The amplitude of this function is high when the masses are in opposite phases of their oscillations.

Main goal of this experiment was to make graphs of both angles and analyze how these could look like for different phases of move.