

# Hyperfine Structure of Rubidium

Phys 1560 & 2010 – Brown University – October 2010

## Purpose:

- To measure hyperfine splitting of atomic energy levels in  $^{87}\text{Rb}$ .
- To become familiar with optical techniques
- To become familiar with Michelson interferometry and measurement calibration

## Safety

The EOS laser which you will use in this experiment is a class IIIb laser with output in the mW range; the laser won't hurt your skin, but it could destroy your vision. Optical tables are kept at waist level, with the idea that if all optical beams are kept in the horizontal plane, then the probability of a laser beam inadvertently burning your eye is dramatically reduced. When doing this experiment do not bend down to “see better” because if you do, you may be **permanently blinded**. *To prevent inadvertent reflection of the laser beam into your eye, remove all watches, jewelry, and reflective clothing.*

According to 2007 ANSI standard Z13, for the range of wave lengths that include 780 nm (the approximate wave length used in this experiment) no laser safety goggles are required below 5 mW. However, since this wavelength is invisible and will not invoke the blink reflex, ANSI requires the no goggles threshold to be at a power level lower than that required to damage the eye in 10 sec. - 0.5 mW (~76 mA at 780 nm). Most of the setup for this experiment can be done below 0.5 mW. But, to actually perform this experiment the power will need to be ~87 mA (~3.6 mW). If the laser power is equal to or above 0.5mW (~76mA) you must wear the appropriate laser safety goggles.

## Doppler-Free Spectroscopy

In this experiment you will use a technique known as saturation-absorption spectroscopy to study the hyperfine structure (hfs) of rubidium. This particular method is designed to overcome the limitations imposed by the Doppler-broadening of spectral lines while avoiding the need to work at low temperatures.

## Theory

In its ground state rubidium (Rb) has a single 5s electron outside a closed shell; the principal structure of the excited states is, therefore, similar to that of hydrogen. The first excited state is split in two by the spin-orbit (s-o) interaction, and each of these levels is split further by the hyperfine interaction. Neglecting relativistic and reduced-mass effects, the Hamiltonian consists of four terms:

$$H_{kin} = \frac{p^2}{2m}$$

$$H_{coulomb} = -Z_{eff} \frac{e^2}{r}$$

$$H_{s-o} = f(r)L \cdot S$$

$$H_{hfs} = \alpha J \cdot I + \beta \frac{[3(J \cdot I)^2 + \frac{3}{2}(J \cdot I) - I(I+1)J(J+1)]}{2I(2I-1)J(2J-1)}$$

Here  $\mathbf{J}$ ,  $\mathbf{L}$  and  $\mathbf{S}$  have their usual meanings, and  $\mathbf{I}$  is the operator for nuclear magnetic moment. The Coulomb potential is assumed to be spherically symmetric, and screening effects are accounted for via  $Z_{eff}$ . The function  $f(r)$  is roughly proportional to  $r^{-3}$  (spin-orbit coupling is discussed in Griffiths' *Introduction to Quantum Mechanics* Ch 6.3.2, and the hyperfine splitting is discussed in Ch 6.5).

The hyperfine interaction is the sum of two terms, multiplied respectively by  $\alpha$  and  $\beta$ . The first term is analogous to the spin-orbit term in that it results from considering the magnetic dipole interaction  $-\mu_n \cdot B_e$ . In this case, however,  $\mu_n$  refers to the nuclear magnetic moment (isospin). Such spin-spin coupling is the dominant contribution to the hyperfine splitting (hfs). The constant  $\alpha$  is called the magnetic hfs constant, and has units of energy. The second term is more complex, arising from the quadrupole moment of the nucleus. Not surprisingly,  $\beta$  is called the quadrupole hfs constant. Rubidium has no permanent dipole moment.

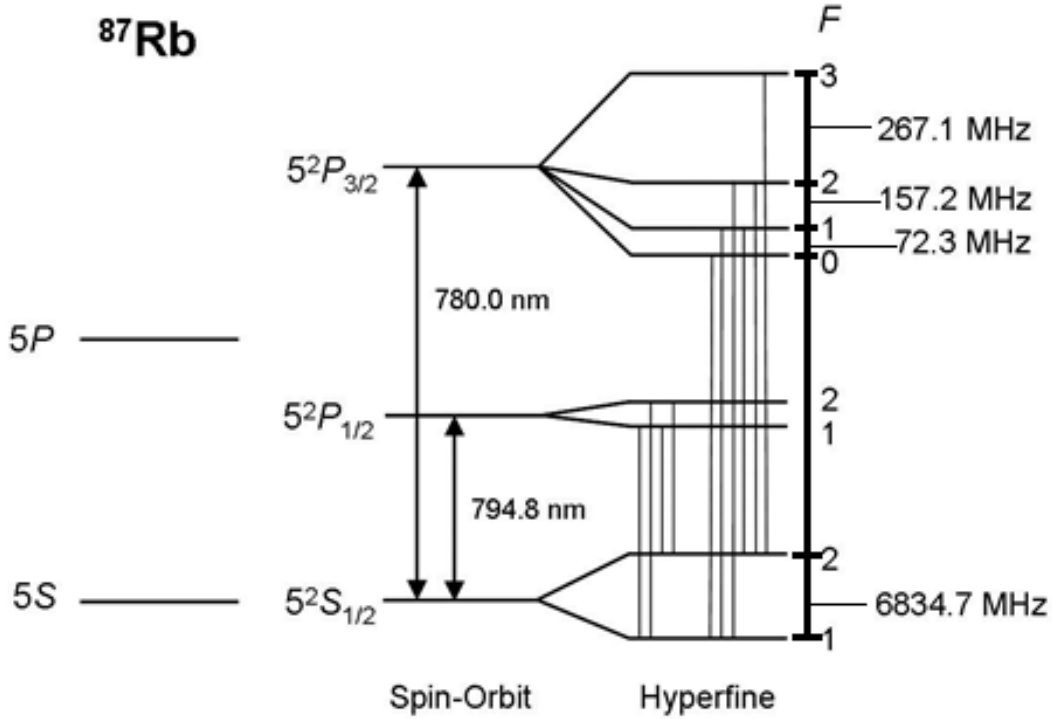
Rubidium has two naturally occurring isotopes,  $^{87}\text{Rb}$  (28% abundant,  $I=3/2$ ) and  $^{85}\text{Rb}$  (72% abundant,  $I=5/2$ ). We will study the structure associated with the  $5^2P_{3/2}$  to  $5^2S_{1/2}$  transitions in  $^{87}\text{Rb}$ , as shown in Figure 1. The total angular momentum of a state is designated by  $\mathbf{F}$ , where  $\mathbf{F} = \mathbf{J} + \mathbf{I}$ . From this you can easily verify that each of these levels is split as shown. However, due to electric dipole selection rules, only transitions with  $\Delta S=0$ ,  $\Delta J=0, \pm 1$  are allowed (Griffiths Ch 9.3.3). From this we conclude that hfs transitions must have  $\Delta F = 0, \pm 1$ .

The frequency spacings between adjacent levels as shown in Figure 1 can be calculated by rewriting the dot product  $\mathbf{J} \cdot \mathbf{I} = (\mathbf{F}^2 - \mathbf{J}^2 - \mathbf{I}^2)/2$ . In an eigenstate of these operators we have, for example,

$$F^2|k\rangle = F(F+1)|k\rangle$$

where  $k$  denotes the collective index. Thus we see that in such a case

$$J \cdot I = \frac{1}{2}[F(F+1) - J(J+1) - I(I+1)] \equiv \frac{C}{2}$$



**Figure 1.** Energy level diagram for  $^{87}\text{Rb}$ . The frequency spacing between adjacent levels is shown for the hyperfine splitting.

Then, from  $H_{hfs}|k\rangle = E_{hfs}|k\rangle$  we find

$$E_{hfs} = \alpha \frac{C}{2} + \beta \frac{\frac{3}{4}C(C+1) - I(I+1)J(J+1)}{2I(2I-1)J(2J-1)}$$

which gives the perturbed energy—the size of the deviation from the unperturbed state. This is often written in terms of the frequency:

$$\nu_{hfs} = A \frac{C}{2} + B \frac{\frac{3}{4}C(C+1) - I(I+1)J(J+1)}{2I(2I-1)J(2J-1)} \quad (1)$$

The accepted values of the coefficients  $A$  and  $B$  are tabulated below in Table 1. From the above equation, you should be able to convince yourself that the quadrupole term must vanish for  $5^2S_{1/2}$ .

Units are MHz	85Rb	87Rb
A for $5^2S_{1/2}$	1011.91	3417.34
A for $5^2P_{3/2}$	25.01	84.85
B for $5^2S_{1/2}$	—	—
B for $5^2P_{3/2}$	25.90	12.52

Table 1

**Exercise:** use the above equation to verify the frequency spacing for several hyperfine levels. If you take the time to do the same for the more abundant  $^{85}\text{Rb}$ , you will see why we focus our attention on the less abundant isotope.

### The Doppler Broadening of Absorption Spectra

If we direct a laser at a collection of Rb atoms, photons cause transitions between states whenever the energy is an exact match; this absorption of energy can be measured to observe the energy transitions. At zero temperature, each energy level has a slight fuzziness given by the uncertainty principle  $\Delta E \Delta t \geq \frac{\hbar}{2}$  where  $\Delta t$  is the lifetime of the state. Therefore, an atom can absorb light over a small range of frequencies, called the natural line width of the transition. However, atoms at room temperature are in constant, random thermal motion. This means that a substantial fraction of the atoms will see either red-shifted or blue-shifted photons, resulting in a further broadening of the line width. If we consider only motion along the axis defined by the beam of light, then the Doppler-shifted frequency is given by

$$\nu_L = \nu_0(1 \pm V/c).$$

Where  $\nu_0$  refers to the resonance frequency when the atom is at rest,  $\nu_L$  to the light frequency, and  $V$ . It is possible to estimate  $v$  from the equipartition theorem, but a more useful expression can be obtained by considering the Maxwell velocity distribution of particles in a classical ideal gas:

$$P(V)dV = \sqrt{\frac{m}{2\pi k_B T}} e^{-mV^2/2k_B T}$$

If we re-write the Doppler shift as

$$V = c(\nu_L - \nu_0)/\nu_0.$$

and substitute this expression into the Maxwell distribution, we find the probability that an atom will see photons of a given frequency is

$$P(\nu_L)d\nu_L = \frac{\delta}{\sqrt{\pi}} e^{-(\nu_L - \nu_0)^2/\delta^2} d\nu_L.$$

The expression is a Gaussian, given in terms of the line width parameter  $\delta = \frac{\nu_0}{c} \sqrt{\frac{2k_B T}{m}}$ . It is conventional to specify the width of a given transition by the full-width at half maximum (FWHM), which is given by

$$\Delta\nu = 2\delta\sqrt{\ln(2)}.$$

At 300K, the 780 nm transition ( $\nu_o = 3.84 * 10^{14} Hz$ ),  $\delta = 305.61 MHz$ , and  $\Delta\nu = 508.88 MHz$ . Compare this to the natural linewidth of the transition,  $\Delta\nu_{nat} = \frac{1}{2\pi\Delta t} = 5.68 MHz$ . The Doppler broadening of the frequencies seen by the Rb at room temperature is two orders of magnitude larger than the natural linewidths, meaning the hyperfine structure associated with the 780 nm transition is lost in the Doppler spread. Clearly a way around this must be found in order to observe the hyperfine transitions.

### Saturated Absorption Spectroscopy

Saturated absorption spectroscopy is the name given to a very clever trick which overcomes the limitations imposed by Doppler-broadening. The mechanism by which it works depends critically on the rate of optical pumping and the lifetimes of the hyperfine states, and is quite involved. With this in mind we will proceed from a more empirical point of view.

Figure 2 shows the optical apparatus. When an on-resonance laser beam passes through the sample, power will be absorbed by atoms in the sample and a detector picking up the beam on the other side would register a Doppler-broadened absorption spectrum as discussed above. Saturation absorption spectroscopy bypasses the Doppler broadening through the use of two separate beams. The higher-intensity pump beam saturates the higher state, so as the probe beam passes through in the opposite direction, there is a decreased number of atoms in the zero-velocity ground state to absorb its photons. The decreased power loss resulting from reduced absorption of photons from the probe beam by atoms in the sample is measured at the detector.

The beauty of this method is that because the beams travel in opposite directions, only atoms which are at rest relative to both beams are in resonance with both. Thus the absorption signal detected by the probe should exhibit a small, narrow spike at the location of the “zero temperature” line, and its width should approach the natural line width.

### Cross-Over Resonances

You will also need to look out for “cross-over” resonances, a phenomenon peculiar to saturation absorption spectroscopy. These resonances only occur when two (or more) different transition frequencies are within the Doppler-broadened frequency distribution of the laser. Under these conditions, at a frequency halfway between two transitions, there will be some nonzero velocity with respect to the propagation of the laser for which the Doppler shift is just right to induce a transition. Atoms moving with this velocity will saturate one of the two excited states—let’s say state A.

However, in this scenario the other transition—to state B—is excited by the exact opposite Doppler shift. In other words, the transition to state B will be stimulated for atoms moving with the same speed but in the opposite direction with respect to the direction of propagation of the beam. But the probe beam enters the sample in the opposite direction, so it will be these same atoms just excited by the transition to state A which will now see the correct Doppler shift in photons from the probe beam to transition to state B.

Because of the depleted number of atoms in the ground state due to the saturation of state A by the pump beam, there will be a decrease in absorption of photons from the probe beam. This is observed as an increase in the power reaching the detector (and corresponding voltage spike on the oscilloscope,) just like in the case where the laser is precisely on the transition frequency—a sort of false positive. Cross-over resonances are often even stronger than the desired “zero temperature” resonances! These cross-over resonances, however, allow for another method of measuring the ground state energy.

### **Other Notes**

There are other factors which influence line width. These interactions are given the collective title of “pressure broadening”, and are represented by the expression  $V_n = C_n/r_n$  where  $n = 1$  and  $n = 2$  are associated with the Stark effect due to interactions with hydrogen or helium (these probably have a vanishingly small contribution in our experiment). Most relevant are  $n = 3$  (dipole interactions) and  $n = 6$  (van der Waals interactions). It is not expected that you will reach any comprehensive conclusions about these effects, though you should be able to estimate their magnitude.

Now that you have considered the exercise and the questions, you should be convinced that the  $F = 3, 2, 1 \leftrightarrow 2$  transitions should be resolvable under the conditions of this experiment. A central goal is to measure the frequency spacing of these levels. Note that the  $F = 2, 1, 0 \leftrightarrow 1$  transitions are also resolvable, but the optical pumping (the intensity of the pump beam relative to the intensity of the probe) reduces them to marginal levels.

### **Equipment**

- EOS 2010 External Cavity Diode Laser with power supply
- IR viewing scope and IR viewing card
- Oscilloscope
- Function Generator
- 3 mirrors, one regular beamsplitter, 1 microscope slide beamsplitter, and 2 photodetectors
- assorted posts, clamps and magnetic bases
- assorted coaxial cables

### Diode Laser

For optimal stability the laser must warm-up for an extended period. It is accepted practice to leave the laser on in “sleep mode” when not in use.

Inside the laser head, the output of the diode module is feed to a cavity feedback system which consists of a “tuning mirror” and a “grating module”. The position of the tuning mirror is adjustable and its position relative to the grating module sets the output wavelength of the laser. The position of the tuning mirror (and the wave length of the laser) can be manually adjusted with a knob on the rear of the laser head. A piezoelectric crystal attached to the tuning mirror can be used to scan the laser wavelength. The connection to the piezoelectric crystal is on the front of the power supply.

Since the wavelength of the laser is set by the feedback cavity, it is important to prevent

laser light external to the laser head from scattering back into the cavity. If this is allowed to happen, the laser will become highly unstable.

All of the above is more thoroughly explained in the EOS manual <https://wiki.brown.edu/confluence/download/attachments/29406/Diode+Laser+Manual.pdf?version=1&modificationDate=1260814471000>. Pages 2–4 (introduction and safety), 13–19 (operation) and 27–30 (theory of external cavity operation) will be particularly helpful.

Note: The only adjustment you should make to the laser head is the position of the tuning mirror. Do not open the lid to the laser head.

Only low voltage, low frequency modulations are required in this experiment. Keep the scan frequency below 25 Hz and the peak-to-peak voltage below 14 V.

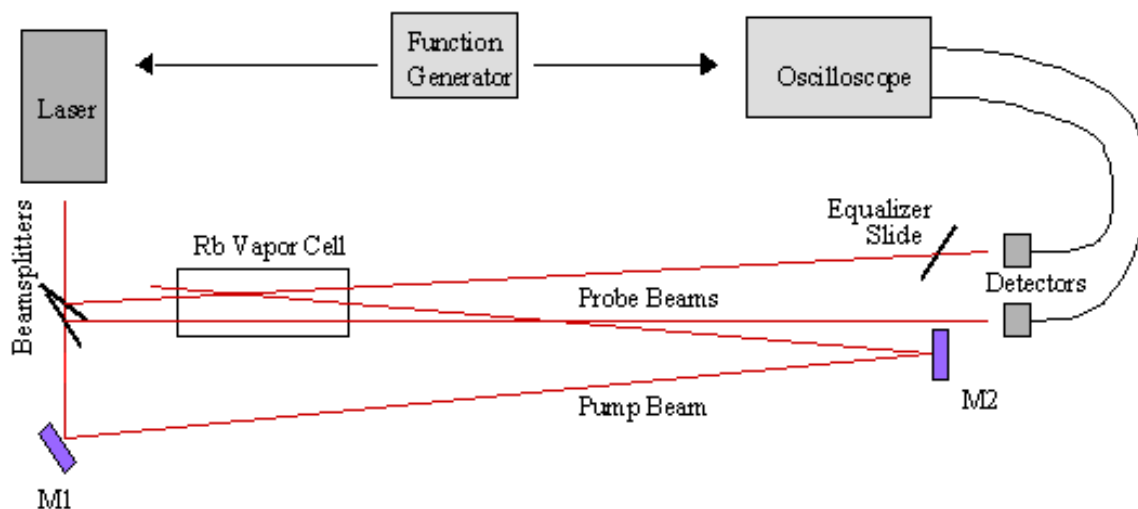


Figure 2. Experimental setup. Note that the pump beam only crosses one of the two probe beams inside the Rb cell.

**Procedure** (Note: The references contain a comprehensive treatment of this technique.)

1. Be safe. If the current is greater than 76mA; wear safety goggles, take off watches, rings, etc. You know the drill.
2. Do not touch the optical surface of any optical component. Make sure the center of all optical surfaces is at approximately the same height. Adjust each optical surface so the laser light hits near its center and the reflected or transmitted light is kept parallel with the table top. This will make working with optics much easier and improve your chances of success.
3. To find the transition frequency, place the rubidium cell about 4 inches away from the laser directly in front of the beam. Set the laser current to ~87 mA. With the lights off, use the IR goggles to observe the Rb cell. With the adjustment knob on the back of the laser head, slowly vary the laser's frequency around 780 nm. Do not vary the frequency more than +/- 2 nm. Resonance in the Rb cell should be obvious. What do you observe in the cell at resonance? Be sure to turn off and

- cap the IR goggles before turning on the room lights.
4. Position a detector on the other side of the Rb cell to detect the laser light. Connect the detector's output to an oscilloscope input. Connect the function generator to the piezo input on the laser power supply and to a different oscilloscope input. Also connect the function generator's sync out to the Aux In on the oscilloscope and set the oscilloscope to trigger on Aux In. Set the function generator to triangle wave at 1 Vp-p and 1 Hz. Observe the relationship between the triangle wave and the detector output on the oscilloscope. Adjust the function generator's frequency and voltage to get four distinct peaks in the detector signal for each ramp up or down of the triangle wave (the fourth peak is small). The maximum modulation input voltage at various frequencies are given on page 26 of the EOS laser manual but you will not need to go above 25 Hz or 10 Vp-p. With the adjustment knob on the back of the laser head, slowly vary the laser's frequency to center the structure in the detector signal to the center ( $V=0$ ) of each ramp of triangle wave -  $V=0$ . If the detector signal jumps around strangely, increase the intensity of the laser current.
  5. Set up the configuration in figure 2. Position each optical component such that the light interacts with the center of its active element (mirror etc.). Adjust the beamsplitters, M1 and all following components such that the light leaving them is parallel with the table top.
  6. Make sure the pump beam only crosses one of the probe beams inside the Rb cell. Make sure all laser beams enter and exit the cell through the optically flat ends of the cell without contacting the cylindrical walls of the cell. Make sure no stray reflected beams enter the cell. And finally, make sure no laser light goes back into the laser cavity.
  7. Once the optics are properly aligned, the hyperfine structure will be visible on the oscilloscope. As long as the pump beam only interacts with one of the probe beams inside the Rb cell, the signals from the two detectors can be subtracted to remove the Doppler-broadened background and allow direct observation and measurement of the hyperfine structure. Record your data. Note: The equalizer slide is to reduce the intensity of the probe beam coming from the first front surface reflection in the beamsplitters.

### Calibration

To calibrate the frequency shift of the laser as a function of applied voltage, construct the Michelson interferometer shown in Figure 3. The recombined beams will have a phase difference

$$\phi_1 - \phi_2 = \frac{2\pi}{\lambda} (2L_1 - 2L_2) = \frac{4\pi\nu}{c} (L_1 - L_2).$$

Since the wavelength is being swept continuously, we actually have

$$\Delta(\phi_1 - \phi_2) \propto \Delta\nu.$$

The intensity is a maximum when the phase difference is an even multiple of  $2\pi$ ,



therefore the maxima occur at regular frequency intervals given by

$$\Delta\nu = \frac{c}{2(L_1 - L_2)}$$

The frequency separation between two adjacent peaks in the detector output occurs at this frequency interval. Use the average change in ramp voltage between adjacent peaks in the detector output to calibrate the frequency shift to change in piezo input voltage.

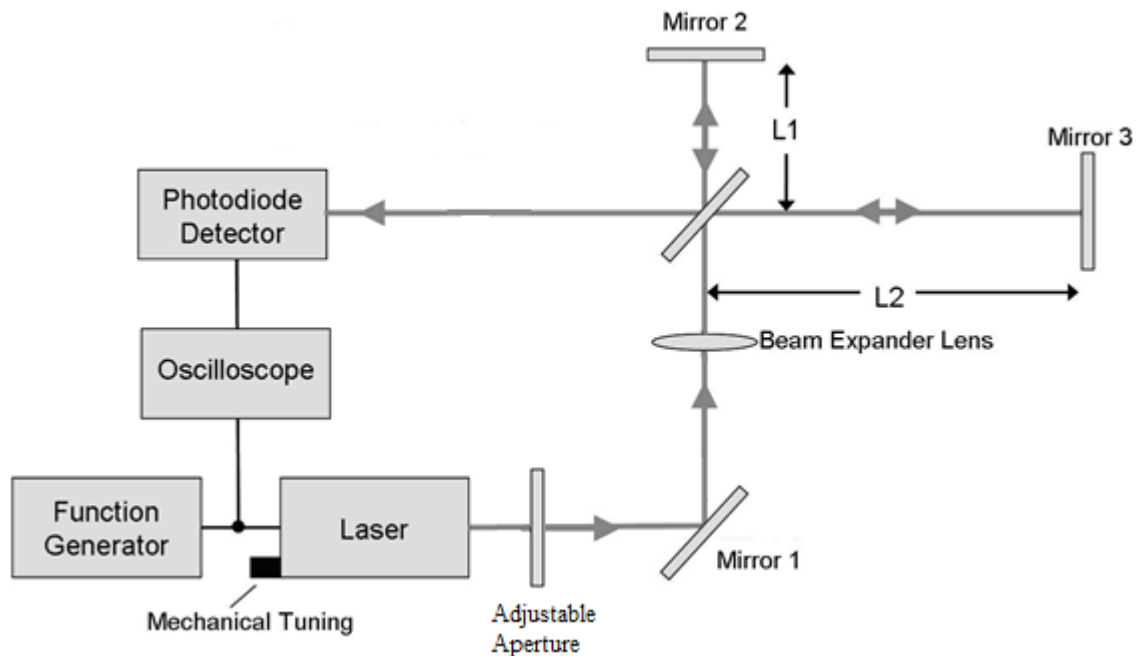


Figure 3. Interferometer configuration

### Interferometer

1. Be safe. If the current is greater than 76mA; wear safety goggles, take off watches, rings, etc. You know the drill.
2. Do not touch the optical surface of any optical component. Make sure the center of all optical surfaces is at approximately the same height. Adjust each optical surface so the laser light hits near its center and the reflected or transmitted light is kept parallel with the table top. This will make working with optics much easier and improve your chances of success.
3. Excluding the beam expander and photodetector, setup the optical configuration shown in figure 3. Set the difference in path length to ~30 cm. Set the opening in the adjustable aperture as small as possible without blocking too much laser light.
4. Observe and adjust the two elongated spots on the wall. Using the adjustment knobs on the mirrors 2 & 3, position the two spots directly on top of each other. Note the location of the spot on the wall. The current can be below 76 mA.
5. Place the beam expander in the configuration. Make sure the beam passes through the center of the lens on its optical axis and that the location of the center of the now larger spot on the wall is very near its original location.
6. Set the current to above 86.5 mA and set the piezo frequency to ~0.1 Hz.

- Carefully adjust mirrors 2 & 3 until a relatively small oscillating bulls-eye appears on the wall – this is the interference pattern.
7. Place a card in front of the bulls-eye and carefully move the card to the intended location of detector. If the bulls-eye disappears and this new distance, carefully readjust the mirrors until it reappears.
  8. Once the bulls-eye has been created at a location suitable for the detector, place the detector at that location. Carefully adjust the mirrors until the center of the bulls-eyes is on the center of the detector.
  9. Connect the output of the detector to the oscilloscope and set the function generator back to the value you used during data collection. Figure 5 shows an example of what to expect.
  10. Collect data.

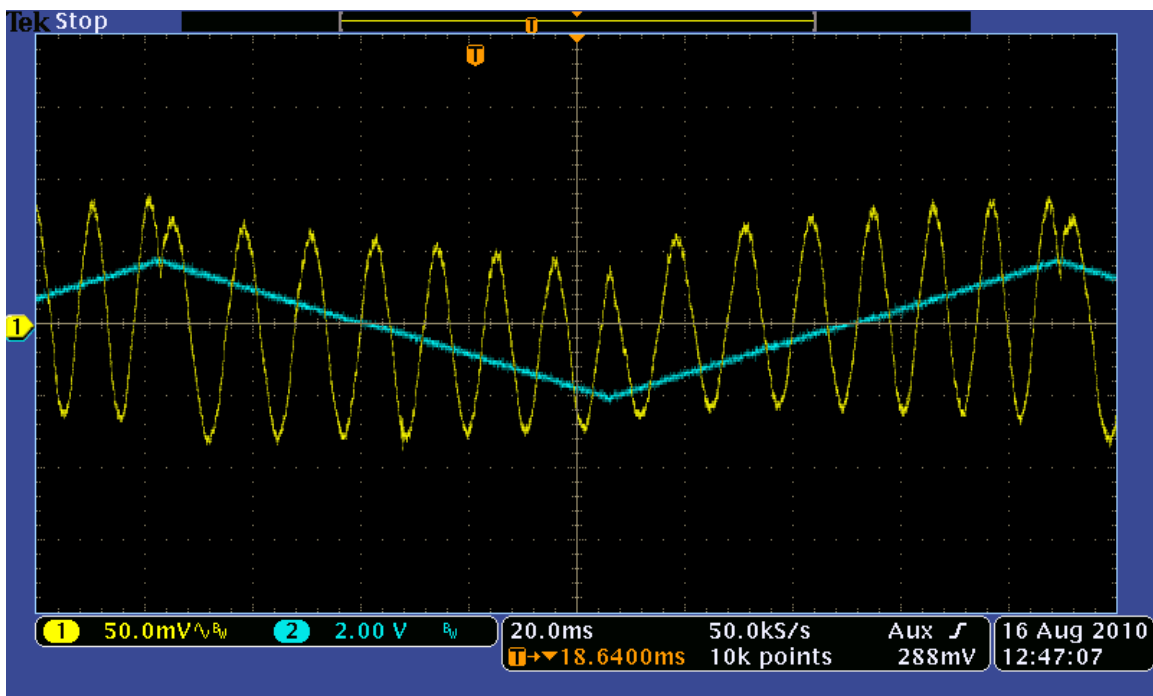


Figure 5. Typical interferometer output. Note the behavior of the signal near the discontinuities of the ramp.

### References

- [1] Baym, Lectures on Quantum Mechanics, Perseus Books, 1969
- [2] Corney, Atomic and Laser Spectroscopy, Clarendon Press, 1977
- [3] Advanced Optics Laboratory, Doppler-free saturated absorption spectroscopy: Laser spectroscopy.  
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