Chaotic behavior in electrical circuits

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Introduction

Chaotic behavior is usually thought of in terms of tangible or visible systems, like fluids and weather systems. Electrical circuits, however, can also exhibit chaotic behavior. The aim of this project is to simulate two electrical circuits – an autonomous relaxation oscillator and Chua's circuit – with known capacities for chaotic behavior. The simulation is designed to assist the user in locating parameters and initial conditions that create chaos.

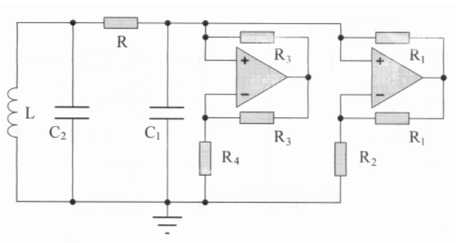

Fig. 1 Chua's circuit

The Simulation

The user can simulate either Chua's circuit or an autonomous relaxation oscillator, shown in figures 1 and 2, respectively. The simulation is loaded with several preset initial conditions and circuit parameters that create interesting and chaotic behaviors, and the user may also tune these variables individually to explore interesting behaviors.

In addition to running the selected circuit, the user can also study how the long-term behavior of the circuit varies with the value of a single circuit element. Figure 5 demonstrates this capability on Chua's circuit.

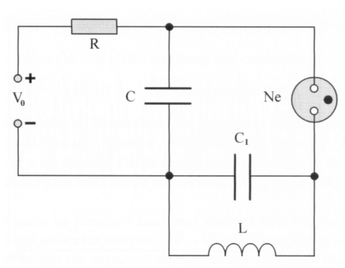

Fig. 2 An autonomous relaxation oscillator

Results and Conclusions

While the autonomous relaxation oscillator is elegantly simple and offers a nice introduction to chaotic electrical circuits, the added complexity of Chua's circuit offers greater promise for in-depth study. Our initial exploration has located a band of chaotic behavior on R from 0.682 to 0.703 with the circuit parameters seen in Fig 3. The identification of parameters that lead to chaos is presently laborious and somewhat dependent on lucky selection, but planned developments will automate much of the search procedure.

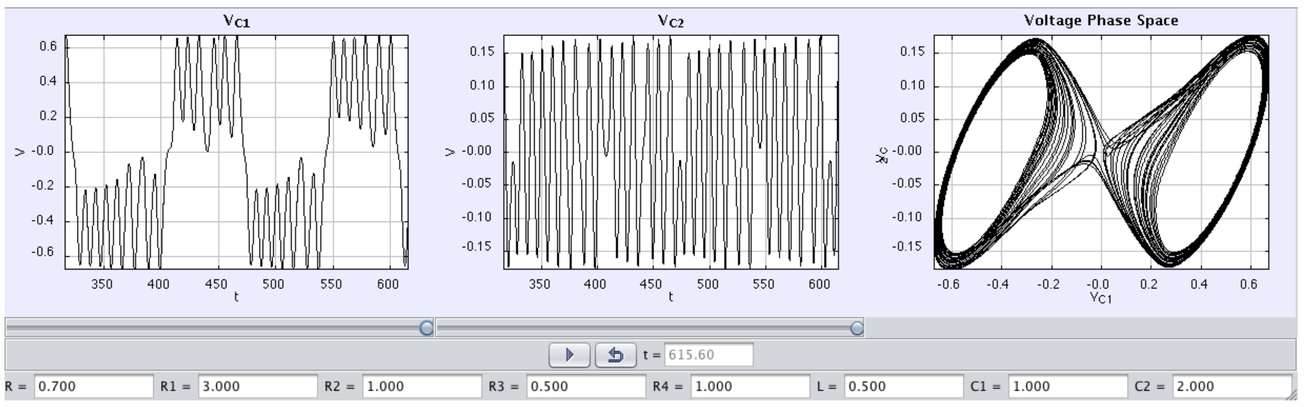

Fig. 3 A simulated run of Chua's circuit with the listed initial conditions

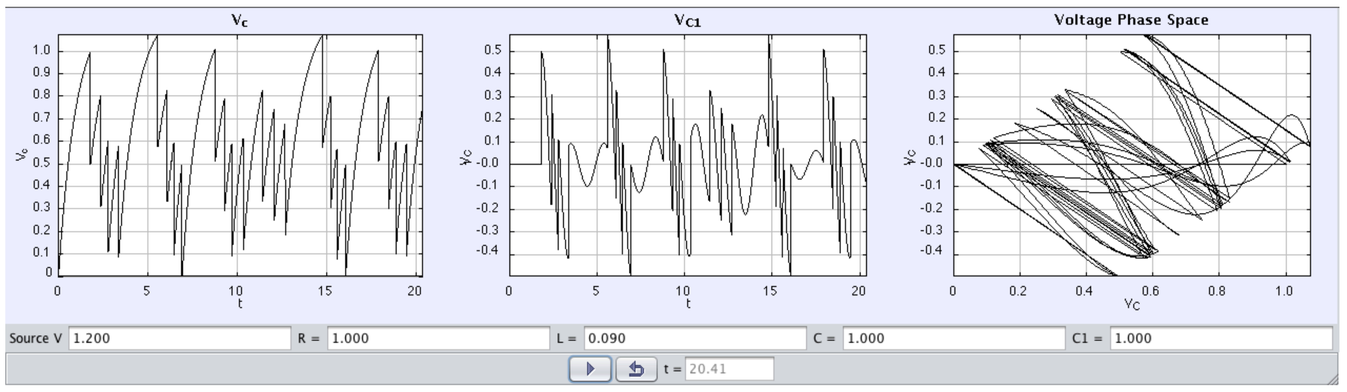

Fig. 4 A simulated run of the autonomous relaxation oscillator. The firing of the neon lamp (labeled Ne in Fig. 2) is apparent in the sharp corner points in all three plots.

Development

A fully-developed simulator will include more robust features for identifying and exploring potential regions of chaotic behavior. These include the capability to track long-term voltage behavior against all circuit parameters and against two parameters at once, which would create a 3-dimensional plot.

Most importantly, the long-term tracker will be refined to plot only voltage minima and maxima. This improvement will allow reliable identification of chaotic behavior, as opposed to the current feature, seen in Fig 5, wherein stable oscillations between two values look nearly identical to chaotic oscillations between the same two values.

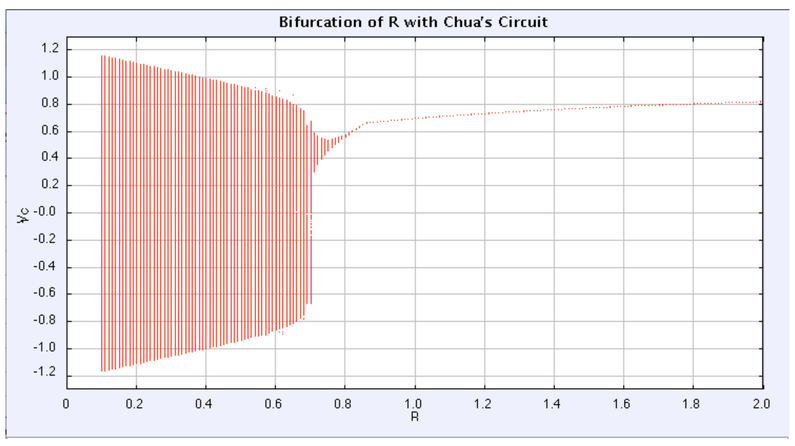

Fig. 5 The long-term voltage values in C in Chua's circuit, plotted against R. Low values lead to periodic oscillation ,and high values lead to a flat, stable voltage, but values between 0.682 and 0.703 produce chaos.

References

[1] *Elegant Chaos: Algebraically Simple Chaotic Flows*, by Julien C. Sprott

[2] Wolfgang Christian's EJS adaption of "An Introduction to Computer Simulation Methods: Applications to Physical Systems," by Harvey Gould, Jan Tobochnik, and Wolfgang Christian

[3] OSP Collection on the ComPADRE Digital Library. http://www.compadre.org/osp/

[4] Easy Java Simulations. http://www.um.es/fem/EjsWiki/