

## HALL EFFECT

### 1. References

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J. P. McKelvey, "Solid State and Semiconductor Physics", Harper and Row, 1966

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E. H. Hall, American Journal of Mathematics, 2, 287 (1879)

### 2. Purpose

This experiment familiarizes you with the electronic transport properties of semiconductors. You also learn to carry out electrical measurements over an extended temperature range between 78 - 430K.

### 3. Introduction

In 1879 E. H. Hall observed that when a magnetic field  $B$  is applied at right angles to the direction of current flow in a conductor, an electric field is created in a direction perpendicular to both.

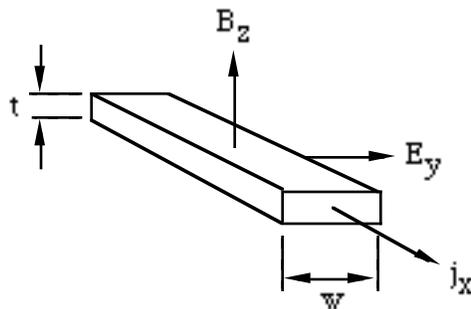


Fig. 1 Coordinate, Field, and Current Definitions in Hall Effect Sample

Intuitively, one may consider the effect to result from the Lorentz force  $F_B$  on the free conduction electrons of charge  $e$  moving along the conductor with a drift velocity  $v_x$

$$F_B = e(v_x \times B_z).$$

Under the influence of this force, the conduction electrons crowd to one side of the conductor, creating an electric field  $E_y$  which generates an opposing force on the conduction electrons

$F_E = eE_y$ . The reaction is stabilized when

$$F_B = F_E = e(v_x \times B_z) = eE_y .$$

Since the current density is  $j_x = env_x$ , the Hall field  $E_y$  is proportional to  $j_x$  and  $B_z$ . The Hall coefficient  $R$  is defined by

$$E_y = R j_x B_z \quad (1)$$

where  $R$  is given in  $\text{cm}^3/\text{Coulomb}$  when  $j_x$  is measured in  $\text{A}/\text{cm}^2$ , and  $E_y$  in  $\text{V}/\text{cm}$  and  $B_z$  in  $\text{Vsec}/\text{cm}^2$  ( $= 10^8$  Gauss). The Hall effect is the most important of the galvanomagnetic effects because it yields values for the charge carrier concentration and the sign of the carriers as well as their mobility when  $R$  values are combined with electrical conductivity ( $\sigma$ ) measurements. The Hall Effect finds applications for magnetic field measuring devices and perhaps in the future for magneto-hydrodynamic power generation.

## 4. Theory

### 4.1 Simple Model for One Type of Charge Carrier

The theory of the Hall effect is particularly simple if one assumes conduction by carriers of a single type with equal charge  $q$  and equal mobility  $\mu$  and with a charge density  $qn$  per unit volume. The Lorentz force on these charge carriers amounts to  $j_x B_z$  per unit volume in the  $-\hat{y}$  direction (see Fig. 1). As a result the carriers are deflected toward  $-\hat{y}$  until an excess charge accumulated on the  $y$ -surfaces generates the Hall field  $E_y$  which just counterbalances the Lorentz force. Equilibrium is reached when

$$q n E_y - j_x B_z = 0 \quad (2)$$

so that the Hall coefficient is

$$R = 1 / qn \quad (3)$$

which yields the sign  $q = \pm e$  and the concentration  $n$  of the charge carriers.

If the charge carriers are electrons,  $R$  should always be negative. Experimentally it has been found that  $R$  is positive for many semiconductors and also some metals. However, the free-electron theory of metals provides no explanation for the presence of positive charge carriers. Positive ions have to be discounted as charge carriers here because they could only move with much smaller mobility. The existence of positive charge carriers (holes) is explained by quantum theory as discussed in the textbooks.

A few more relations: you recall that the conductivity  $\sigma$  is

$$\sigma = j_x / E_x = qn\mu \quad (4)$$

The mobility  $\mu$  is defined by the relation for the drift velocity  $v_d$

$$v_d = \mu E_x \quad (5)$$

and can be calculated from

$$\mu = R \sigma \quad (6)$$

#### 4.2 Hall Effect for Two Types of Carriers

Let us assume both electrons and holes are present with concentrations  $n$  and  $p$ , mobilities  $\mu_n$  and  $\mu_p$ , and current densities  $j_{xn}$  and  $j_{xp}$ , which add up to the total current density

$$j_x = e p \mu_p E_x - e n (-\mu_n) E_x = e (p\mu_p + n\mu_n) E_x \quad (7)$$

In case of two types of carriers, the forces on the carriers are not balanced for each type individually; but surely the current density  $j_y$  in the  $y$ -direction must be zero. Remembering that the charge and the mobility and the drift velocity of the electrons have opposite sign of those of holes we write

$$j_x = e p \mu_p \left[ E_y - \mu_p E_x B_z \right] + (-e) n (-\mu_n) \left[ E_y - (-\mu_n) E_x B_z \right] = 0$$

which yields with Eqs. (1) and (7)

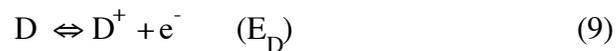
$$R = (p \mu_p^2 - n \mu_n^2) / e (p \mu_p + n \mu_n)^2 \quad (8)$$

The sign of  $R$  depends on that of the numerator, that is, on the relative concentrations and mobilities of electrons and holes.

The above analysis of the Hall Effect is oversimplified in that all charge carriers of one type do not have the same drift velocity as assumed, and hence one should average over the distributions of  $v_d$  for electrons and for holes. This effect increases the value of  $R$  in Eqs. (3) and (8) by a factor  $r$  which is temperature dependent and of magnitude  $1 < r < 2$ . Moreover, the valence bands of most semiconductors really consist of two distinct overlapping bands. As a result of this there are two groups of holes with different mobilities. Any serious analysis of the experimental data has to take these details into account. We neglect them in this experiment.

### 4.3 Temperature Dependence of Carrier Concentrations

The temperature dependence of electrons in a n-type semiconductor is sketched for different donor concentrations in Fig. 2. We shall explain it by referring to the energy diagram of Fig. 3. Here  $E_g$  is the semiconductor band gap ( $E_g = 0.67$  eV at 300K for Ge) and  $E_D$  is the donor ionization energy (about 0.01 eV in Ge).  $N_c$  and  $N_v$  are the effective density of states in the conduction band and valence band, respectively. Below room temperature, in the extrinsic range,  $n$  is governed by the thermal ionization of donors



This reaction cannot yield more electrons than  $N_D$ , hence  $n = N_D$  in the saturation range. At higher temperatures  $n$  rises rapidly because electrons become thermally excited over the gap  $E_g$  from the valence band to the conduction band. Let us calculate the carrier concentrations in the two regimes.

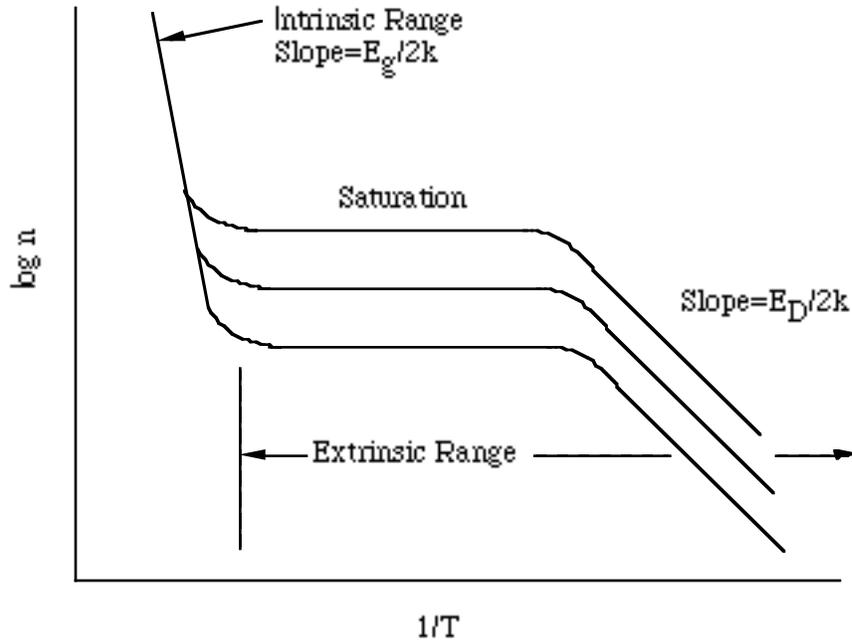


Fig. 2 Temperature dependence of carrier concentration  $n$  in an n-type semi-conductor for three different donor concentrations  $N_D$

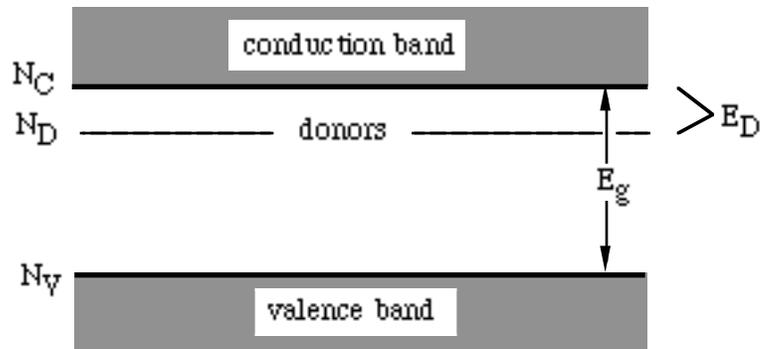


Fig. 3 Schematic diagram of energy levels in n-type semiconductors

Extrinsic Range. The rate of thermal emission of electrons from donor centers to the conduction band is proportional to the concentration of donors still having an electron  $N_D - n$ , and to the effective density of conduction band states  $N_c$

$$k_1 N_c (N_D - n) \quad (10)$$

The capture rate of electrons by donors is proportional to  $n$  and to the concentration of ionized donor states  $N_D^+$ . Because of charge neutrality,  $N_D^+ = n$ , provided the semiconductor does not contain any acceptors. The capture rate is therefore

$$k_2 n^2 \quad (11)$$

At equilibrium these two rates are equal. This yields the law of mass action equation

$$\frac{n^2}{(N_D - n) N_c} = \frac{k_1}{k_2} = K(T) \quad (12)$$

The equilibrium constant  $K(T)$  is given by the Boltzmann factor

$$K(T) = \exp(-E_D/kT) \quad (13)$$

At very low temperatures  $n \ll N_D$  one gets from Eqs.(12) and (13)

$$n = (N_D N_c)^{1/2} \exp(-E_D/2kT) \quad (14)$$

At higher temperatures  $kT > E_D$ , all donors become ionized so that  $n \sim N_D$  in the saturation region.

For a p-type semiconductor containing  $N_A$  acceptors with a hole ionization energy  $E_A$  similar arguments yield

$$\frac{p^2}{(N_A - p)N_v} = \exp(-E_A/kT) \quad (15)$$

Intrinsic Range. The law of mass action for thermal excitation of electrons over the gap  $E_g$  yields

$$np = N_c N_v \exp(-E_g/kT) \quad (16)$$

together with the neutrality condition  $n = p + N_D$  for n-type and  $p = n + N_A$  for p-type semiconductors. The effective densities of states  $N_c$  and  $N_v$  in the conduction and valence band, respectively, are explained in many textbooks:

$$N_{c,v} = 2(2\pi m_{c,v} kT/h^2)^{3/2} \quad (17)$$

where  $m_c$  and  $m_v$  are the electron and hole effective masses. The numerical values for germanium are

$$\begin{aligned} N_c &= 2 \times 10^{15} T^{3/2} \text{ cm}^{-3} \\ N_v &= 1.05 \times 10^{15} T^{3/2} \text{ cm}^{-3} \end{aligned} \quad (18)$$

#### 4.4 Temperature Dependence of the Mobility

The mobility of electrons and holes is limited by scattering with phonons and ionized donors and acceptors. We can roughly estimate the temperature dependences of these two kinds of scattering processes as follows. The mobility is

$$\mu = \frac{e}{m} \tau = \frac{e}{m} \frac{\lambda}{v} \quad (19)$$

where  $\tau$  = mean free time,  $\lambda$  = mean free path, and  $v$  = thermal velocity of carriers, which is proportional to  $T^{1/2}$ .

Phonon Scattering. In this case  $\lambda$  is inversely proportional to the number of phonons which in turn is proportional to  $kT$ . Hence  $\lambda \propto T^{-1}$  and

$$\mu \propto T^{-3/2} \text{ for phonon scattering} \quad (20)$$

The literature values for germanium are

$$\begin{aligned} \mu_n &= 4.9 \times 10^7 T^{-1.66} \text{ cm}^2/\text{Vs} \\ \mu_p &= 1.05 \times 10^9 T^{-2.33} \text{ cm}^2/\text{Vs} \end{aligned} \quad (21)$$

The discrepancy between Eqs. (20) and (21) arises (i) from the fact that below the Debye temperature the number of phonons is not proportional to  $kT$ , (ii) longitudinal and transverse phonons have different coupling coefficients, and (iii) the scattering coefficients depend on the wave number and hence on the energies of the phonons.

Ionized Impurity Scattering. This is Rutherford scattering for which one expects  $\lambda \propto v^4$ . Since  $v \propto T^{1/2}$ , Eq. (19) yields

$$\mu \propto T^{3/2} \text{ for ionized impurity scattering} \quad (22)$$

This describes the observations quite well. Ionized impurity scattering limits the mobility at low temperatures. It probably is negligibly small in the sample of this experiment because the sample contains only a relatively small acceptor concentration.

## **5. Equipment**

### **5.1 Important Notes**

The sample temperature must not exceed 160°C because the leads are attached with soft solder. Do not attempt to remove the germanium sample from its holder.

### **5.2 Sample**

The semiconductor sample is a germanium single crystal which contains some gallium impurities. It therefore shows "extrinsic" hole conduction at low temperatures. At elevated temperatures electrons are excited from the valence band to the conduction band. Even though  $p > n$ , electrons dominate the conduction at high T because of their higher mobility.

The dimensions of the sample and the separation of the electrical contacts are posted next to the experimental setup. Do not try to remeasure these parameters yourself because of the danger of damaging the sample or the delicate leads.

### 5.3 Cryostat and Temperature Measurements

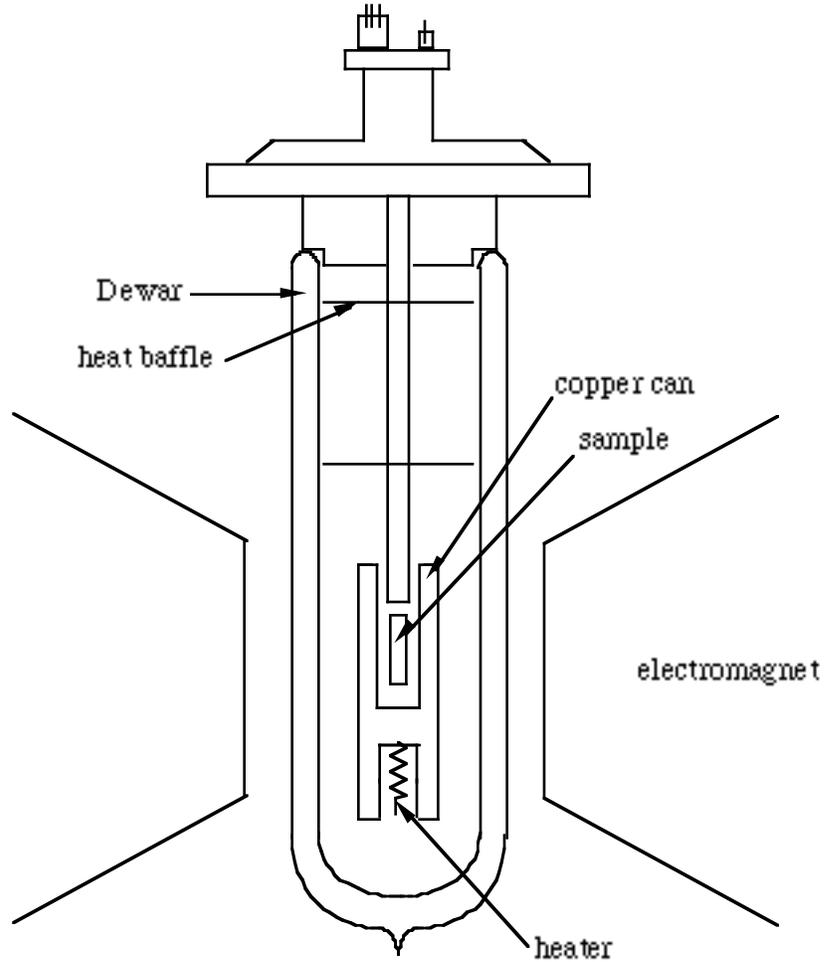


Fig. 4 Hall Effect Sample Holder Assembly

The experimental setup is shown in Figure 4. The sample is mounted inside a copper can. At the bottom of the copper can is a  $130\Omega$  heater. Next to the sample is a copper-constantan thermocouple for temperature measurements. All electrical leads are attached to the connectors on top of the apparatus. A Dewar surrounds the lower part. The cryostat containing the germanium sample can be rotated in a fixed magnetic field either by any desired angle read on the dividing circle or, after fixing the locking screw, by  $180^\circ$ . In addition, the whole apparatus can be moved

horizontally such that the sample is essentially in a field free region. The purpose of these degrees of motional freedom is to permit measurements in both magnetic field directions and at zero field without changing the current in the electromagnet.

The reference junction of the copper-constantan thermocouple is placed in the water just above the ice cubes. Only at that place is the temperature  $T = 0^\circ \text{C}$ . At the bottom of the dewar used for the reference junction the water has  $T = 4^\circ \text{C}$  because this is the temperature at which water has its highest density. Use distilled water and distilled water ice cubes when you really care about the  $T = 0^\circ \text{C}$  accuracy of your reference junction.

#### 5.4 Electrical Measurements

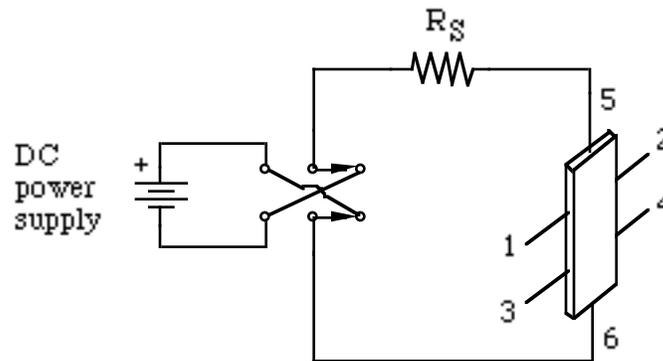


Fig. 5 Hall Effect Measuring Circuit

A recommended measuring circuit is shown in Figure 5. Current  $I$  between 3 and 5mA is passed through leads 5 and 6 of the germanium sample. The current can be determined by measuring the voltage drop across a standard resistor  $R_s$  with a high impedance voltmeter. The direction of the current can be reversed. Note: currents exceeding  $I = 4\text{mA}$  cause some heating in the sample which is noticeable as slow drifts of the voltage readings in the temperature regions where  $\rho$  and  $R$  change rapidly with  $T$ .

The sample leads 1 and 2 are for Hall voltage measurements and leads 1 and 3 for measurements of the resistivity  $\rho$  and the magneto resistance

$$\text{magneto resistance} = (\rho_B - \rho_o) / \rho_o$$

where  $\rho_B$  and  $\rho_0$  are the resistances with and without magnetic field, respectively.

The voltage  $V_{12}$  between leads 1 and 2 is in general not equal to the Hall voltage  $V_H$  but contains four contributions:

$$V_{12} = a + bI + cIB^2 + RIB/t \quad (23)$$

The first term is a thermopower voltage which appears when there is a temperature gradient in the semiconductor. The second term is finite because it is practically impossible to solder leads 1 and 2 precisely opposite each other. Any displacement in the x-direction yields a voltage drop which is proportional to the current I. The third term comes from the quadratic dependence of the resistivity on the magnetic field, in other words, it is the magnetic field induced change of the second term. The fourth term in Eq. (23) is the true Hall voltage  $V_H$ .

Since  $V_H$  is linear in B it can be separated from the other terms in Eq. (23) by measuring  $V_{12}$  with positive and negative B and averaging in the following manner:

$$V_H = \frac{1}{2} [V_{12}(+B) - V_{12}(-B)]$$

## 6. Procedure

In order to do any experiment well one has to repeat it several times. At first one gets a feel of the needed accuracy, some experience of how to adjust the heater current to obtain different temperatures, and some knowledge of the density of data points needed in the different temperature ranges.

The difficulty with this experiment in the student laboratory is the lack of time to carry out this exploratory work. For this reason you will be given suggestions which should alleviate some frustrations.

First you should know that the Hall coefficient R and the resistivity  $\rho$  change only slowly with temperature between 78K and 200K but very rapidly between 200K and 430K. Adjust the density of data points accordingly. Moreover R changes sign from positive for  $T < 270K$  to negative for  $T > 280K$ . At room temperature both R and  $\rho$  are very temperature dependent. Moreover near room temperature or more precisely  $160 < T < 350K$  both electrons and holes

participate in the conduction strongly so that Eqs (7) and (8) are needed instead of the simple Eqs. (3), (4) and (6). Therefore do not start at room temperature but use the following procedure.

6.1 Cool the sample to 78K with liquid N<sub>2</sub>. Here the measurements are very easy: the temperature is constant provided there is enough liquid N<sub>2</sub> in the dewar. You have time to get familiar with the routine of taking data efficiently. Take time to calculate the relevant quantities: the Hall coefficient should be close to  $R=+10^5$  cm<sup>3</sup>/coulomb and resistivity,  $\rho = 3$  ohm cm. Search for mistakes before proceeding if you do not obtain values close to these within a factor 2. Remember that you have to multiply the magnetic field measured in Gauss by  $10^{-8}$  in order to get the magnetic field in units of Vs/cm<sup>2</sup>, then Eq. (1) yields the correct units for R (cm<sup>3</sup>/coulomb)). Rewriting Eq(1) yields

$$R = V_H t/IB \text{ where } t \text{ (cm) is the sample thickness } t = 0.085\text{cm.}$$

6.2 At T = 78K you can measure the Hall coefficient as a function of magnetic field or sample current and you can measure the magneto resistance  $[\rho(B)-\rho(0)]/\rho(0)$ .

6.3 Lay out your measuring routine to be used for the temperatures above 78K. Prepare your notebook so that data writing can be done quickly and reliably following the same routine. Do not measure the magneto resistance at any other T except at 78K. It takes too much time and distracts from the more important quantities R(T) and  $\rho(T)$ . For T > 78K use the highest current I = 4mA and the magnetic field B = 10<sup>3</sup> Gauss. Moreover kept the apparatus in the magnetic field for T>78K. The error you make in using  $\rho(B)$  instead of  $\rho(O)$  amounts to a few percent and is negligible. It will save you a lot of time if you disregard the magneto resistance and leave the apparatus in the magnetic field at the higher temperatures.

6.4 Now you are ready to raise the temperature. Empty the dewar flask and replace it right away to keep the apparatus cool. It is best to leave the dewar in place even above room temperature because air convection is reduced. Do not exceed 160<sup>0</sup>C because all contacts are joined with soft solder. Note : Between 78K and 160K, the resistivity changes by a factor of 5 but R changes very little. Therefore take many more data of  $\rho$  than of R in this T-range.

6.5 To change the temperature it is most convenient to apply for a short time  $\Delta t$  the maximum heater power (voltage = 50V, current = 350mA), then switch the heater power off and wait a few minutes until the change in  $T$  is very slow. Then take measurements quickly. Do not worry too much about a change with time of the third decimal place of the voltages on your digital voltmeter. The large heat capacity of the copper container causes  $T$  to increase only very slowly when the heater is off, slow enough to take data at an essentially constant temperature. You quickly gain experience in estimating  $\Delta t$  of the heat pulse which yields a desired temperature increase.

6.6 For  $T > 160\text{K}$  both  $R$  and  $\rho$  are changing with  $T$ , so always measure both quantities. After switching off the maximum heater power you might have to supply some small heater power to prevent the temperature from decreasing. One can stabilize temperature quite well at  $T > 160\text{K}$  but do not waste time by being too pedantic. Carry the measurements up to  $T = 430\text{K}$  but not higher.

6.7 You need measurements at 30-40 temperature points between 78K and 430K. This should not take longer than 2 hours.

## 7. Presentation of Data and Analysis

7.1 The germanium sample is doped with Ga. Its Hall coefficient is positive at low  $T$  and reverses sign at higher  $T$ . Carry out the data analysis as described by Melissinos. You will notice that the Hall coefficient increases slightly with increasing temperature between  $T = 78\text{K}$  and  $T = 250\text{K}$ . This is surprising because the sample is here in the saturation range where the carrier concentration is constant. This small  $T$ -dependence of  $R$  results from  $T$ -dependence of the Hall factor  $r = R\mu_p$ . In order to find the correct temperature dependence of the mobility  $\mu_p$  in the saturation range, calculate  $\mu_p$  below 250K from  $\mu(T) = R(T = 78\text{K})/\rho(T)$ . Plot for Ge, with  $R$ ,  $\rho$ , and  $\mu$  in units of  $\text{cm}^3/\text{Coulomb}$ ,  $\text{ohm cm}$ , and  $\text{cm}^2/\text{Vs}$ , respectively:

- (1)  $\log_{10} |R|$  against  $1/T$
- (2)  $\log_{10} \rho$  against  $1/T$
- (3)  $\log_{10} \mu_p$  against  $\log_{10} T$  below 250K where current is carried by holes only.

7.2 At liquid nitrogen temperature compare the mobility  $\mu_p$  obtained from the magnetoresistance  $(\rho_B - \rho_0)/\rho_0 = \mu^2 B^2$  with the mobility obtained from  $R/\rho_0 = \mu$ .

7.3 Derive Eq. (17) for free electrons ( $m_c = m$ ), the density of states  $D(E)$  for free electrons and

$$N_C = 2 \int_0^\infty D(E) e^{-E/kT} dE$$

The factor 2 accounts for the spin degeneracy of the electrons.

7.4 In the high T intrinsic range  $N_A$  is negligible compared with  $n$  and  $n \gg p$ . Hence  $|R| \propto \exp(E_g/2kT)$  and  $\rho \propto \exp(E_g/2kT)$ . Determine the energy gap  $E_g$  from the slopes of  $\log |R|$  against  $1/T$  and of  $\log \rho$  against  $1/T$  in the high T intrinsic range. Compare with expected value. You will find your value of  $E_g$  about 0.12eV larger than the literature value. The reason for this is the T-dependence of  $E_g$  which is  $E_g(T) = E_{g_0} - \alpha T$  above  $T = 200K$ . You actually measure  $E_{g_0}$  because Eq. (16) becomes

$$np = N_c N_v \exp(\alpha/k) \exp(-E_{g_0}/kT).$$

7.5 Calculate the acceptor concentration  $N_A$  of the Ge sample and the ratio of  $N_A$  to the concentration of Ge atoms.

7.6 You can determine  $b = \mu_n/\mu_p$ , the ratio of electron to hole mobility very easily from measurements at the temperature  $T_0$ , at which the Hall coefficient reverses sign, that is, where  $R = 0$ . The argument goes as follows: In the saturation range of the extrinsic region, the concentration of holes is  $N_A$  and the resistivity is  $\rho_e = 1/eN_A\mu_p$ . In the intrinsic range the concentration of holes is  $p = N_A + n$  and the resistivity is

$$\rho = 1/e [n\mu_n + (N_A + n)\mu_p] \quad (24)$$

At the temperature  $T_0$ , the numerator of eq.(8) is zero, which means  $p = nb^2$ . With  $p = N_A + n$  this condition becomes  $n = N_A/(b^2 - 1)$ . Hence at  $T_0$  eq.(24) becomes

$$\rho(T_0) = \frac{1}{eN_A\mu_p} \frac{b-1}{b}$$

The extrinsic resistivity extrapolated from lower temperatures to  $T_0$  is  $\rho_e(T_0) = 1/eN_A\mu_p$ . Taking the ratio of this extrapolated resistivity and the measured resistivity at  $T_0$  yields

$$\frac{\rho_e(T_0)}{\rho(T_0)} = \frac{b}{b-1}$$

from which you get the desired quantity

$$b = \frac{\rho_e(T_0)}{\rho_e(T_0) - \rho(T_0)}$$

Determine  $b = \mu_n/\mu_p$  from your data and compare with the literature value.

7.7 Calculate the drift velocity  $v_d$  at  $T = 78\text{K}$  for the electric field you supplied in the x-direction and compare  $v_d$  with the average thermal velocity  $v_{th}$  of a classical electron gas at these temperatures. The expressions used above are valid only when  $v_d < v_{th}$ .