Experiment IV

BALMER SERIES

MASS OF THE DEUTERON

1.   INTRODUCTION

The purpose of this experiment is two-fold: The first objective is to study the hydrogen line spectrum using a diffraction grating spectrometer, and to verify the simple Balmer relation for the hydrogen atom. The experiment can be conveniently divided into four parts: (i) an introduction to the grating spectrometer, (ii) the determination of the grating spacing, (iii) measurement of hydrogen spectral wavelengths, and (iv) determination of the Rydberg constant, using the hydrogen data and the Balmer formula.

The second objective is to determine the mass of the deuteron by measuring the isotope shift between the corresponding wavelengths of hydrogen and deuterium.

2.   HYDROGEN SPECTRUM

2.1   Apparatus

Figure 1 shows a schematic diagram of the spectrometer. The light source is positioned at the collimator slit, whose width can be adjusted. The collimator projects a parallel beam onto the grating. The telescope and cross-hair eyepiece is used to view the diffracted light, while the divided circle and vernier scale allow accurate measurements of angles.

Preliminary adjustments require that the telescope be properly focused, and the grating accurately positioned so that it is normal to the collimator.

2.2   Collimator: At one end of the collimator is a slit illuminated by the light source. The collimator lens collects the light from the slit and forms a parallel beam directed toward the grating. The slit width may be adjusted by turning the slit width adjustment knob.
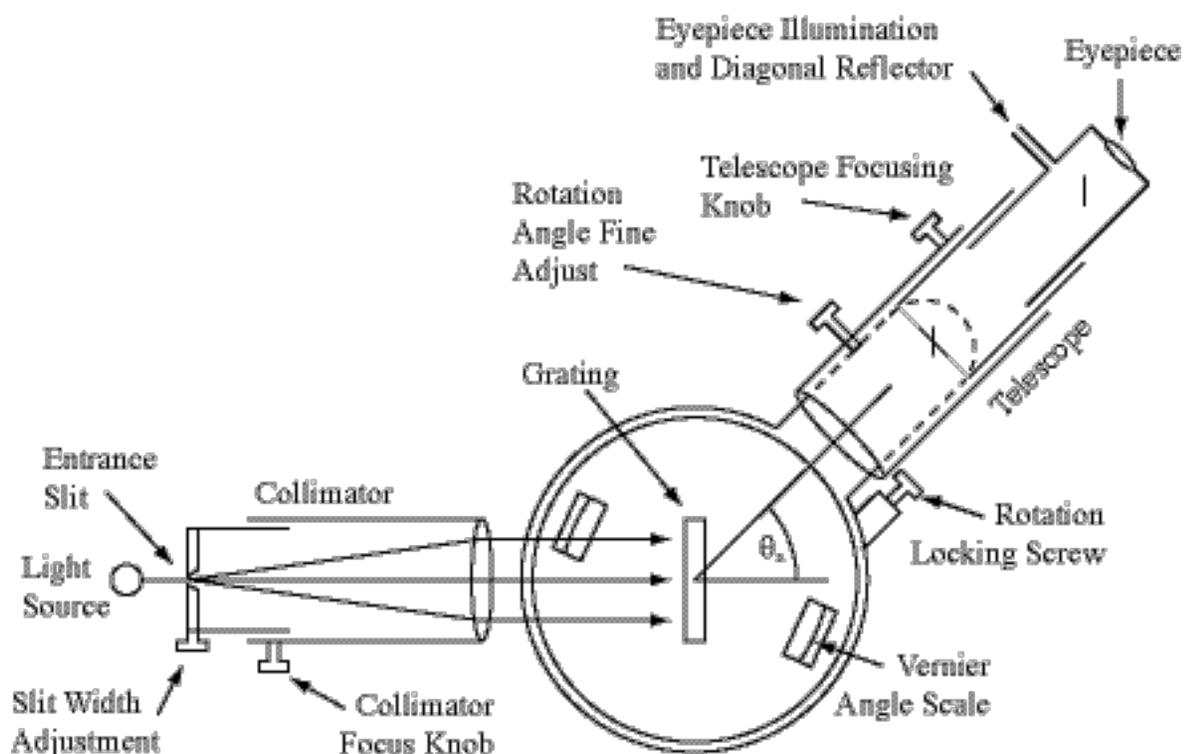
Fig. 1 Grating Spectrometer

2.3 **Grating:** The grating can be rotated and locked by means of knobs on the spectrometer base. The grating equation for normal incidence is given by

$$n\lambda = d \sin \theta \qquad (1)$$

where n is the diffraction order number, d the grating spacing, and θ the diffraction angle, measured with respect to the grating normal. Refer to Jenkins and White, *Fundamentals of Optics*, or other books for a more detailed treatment of this subject.

2.4 **Telescope**: The light exiting the grating arrives at the telescope in parallel bundles, each color arriving at a different angle. Since the rays in each bundle are parallel, the image of the collimator slit is at infinity and the telescope, when focused at infinity, will form a sharp image of the slit at the telescope crosshairs.

2.5 **Divided circle:** Fig. 2 shows the spectrometer vernier angle scale.
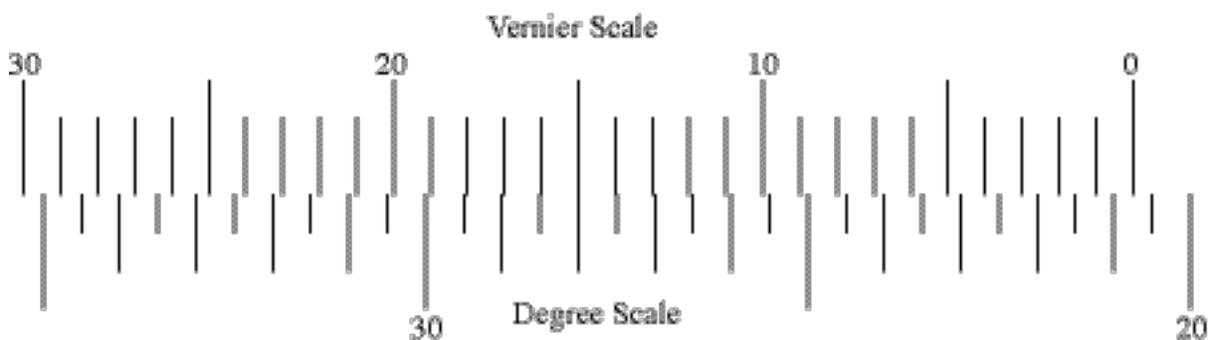
Fig. 2 Angle scale reading 20 degrees, 45 minutes

In order to read the angular position of the telescope on the divided circle, proceed as follows: Read down from the zero mark on the vernier scale to the next smaller line on the degree scale. In the example of Fig. 2 the result would be 20 1/2 degrees or 20 degrees, 30 minutes. Next, find the line on the vernier scale which best coincides with a line on the degree scale. In the example, the best alignment is at 15 minutes. Finally, add the two readings: 20 degrees 30 minutes + 15 minutes = 20 degrees 45 minutes.

Note that there are two angle scales on the spectrometer located 180 degrees apart. They are provided for additional precision. Since it is impossible to machine the divided circle and grating table mount to be perfectly concentric, any eccentricity will yield an angle reading too large on one scale and too small on the other. Therefore, read both and average the results.

3.   Experimental Procedure

3.1   Spectrometer adjustments

   a. Slide the telescope eyepiece in or out to focus the cross hairs.
   b. Place a low intensity mercury lamp at the entrance slit of the collimator.
   c. Move the telescope so that it is co-linear with the collimator, putting the cross-hair intersection point very near the stationary edge of the slit.

d. Examine the locking and slow motion knobs for the grating table and telescope. Note that the slow motion controls function properly only when the locking knobs are tightened. Lock the telescope.

e. Position the grating holder in the central hole of the spectrometer table with the grating facing the collimator. Adjust the grating height.

f. Insert a small mirror into grating holder clips, facing the telescope.

g. Use the eyepiece illuminator to shine light into the hole in the side of the eyepiece while rotating the grating table back and forth. Center the bright disk of light in the telescope field of view.

h. Focus the telescope for the sharpest reflected cross-hair image.

i. Superimpose the vertical crosshairs by rotating the grating table slightly (slow motion control).

j. For finer focus adjustment, move your head from side to side -- no *relative motion* of the two sets of cross-hairs means good focus. (Parallax method.)

k. Remove the mirror and the eyepiece illumination.

l. Focus the collimator slit. Check for parallax between the images of the collimator slit and the cross-hair intersection point.

m. Read the vernier scale for the θ_0 position.

n. To take the data move only the telescope.

3.2   Determining the grating spacing

Referring to the grating equation above, it is clear that we must know the grating spacing d and measure the diffraction angle θ in order to determine the unknown wavelength λ. We shall use a known standard wavelength, measure its diffraction angle and deduce d. The wavelength we shall use is the green line of the mercury spectrum, $\lambda = 546.1\text{nm}$

Using the mercury lamp as the light source, observe the central maximum (zero order) and the first and second order diffraction for the mercury green line on both sides of the zero order.

Note that the grating separates the light into distinct spectral colors. Notice that each color appears as a thin line segment because you are really observing images of the collimator entrance slit. The shape of the slit gives rise to the term spectral "line." Our gratings are *blazed;* that is, the grooves are designed to be asymmetric. Hence, the intensity of the lines on one side is greater than on the other. Each groove has a profile which sends more light to one side than the other, making weak lines more visible. Measure the diffraction angles for $n = 1$ and $n = 2$ on both sides of the zero order. Use these measurements to find $\sin(\theta_n - \theta_0)$ and hence d, the grating spacing. Take several readings to insure precision in the result.

3.3   Measurement of the hydrogen spectrum and the Rydberg constant

Use the hydrogen lamp and measure the four Balmer lines from red to violet. The equation for the Balmer spectrum is

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \tag{2}$$

where $n = 3, 4, 5$ and 6 for the four readily visible spectral lines.

In the Bohr model, assuming that the nuclear mass is much greater than the orbiting electron mass, the Rydberg constant may be derived and is found to be:

$$R_\infty = \frac{e^4}{8\varepsilon_0^2 h^3 c} m_e \quad \text{(S.I. units)} \qquad , \tag{3}$$

where R_∞ is the Rydberg constant, assuming infinite nuclear mass.

Measure the four visible wavelengths of hydrogen and test the functional form of the Balmer formula. Obtain a value for the Rydberg constant R from your data. Compare your experimental value of R to that predicted by eq.(3). Values for the physical constants needed may be found on the inside back cover of this manual.

4.   The Mass of the Deuteron

4.1   Introduction

In reality, the Bohr model of the hydrogen atom is not quite correct. Strictly speaking, the electron does not orbit the "stationary" proton. Rather, both the electron and proton orbit their combined center of mass. The proper treatment of such a system requires that we use the *reduced mass* in place of the electron mass in eq.(3). The reduced mass for hydrogen μ_H (proton-electron system) is:

$$\mu_H = \frac{m_p m_e}{m_p + m_e} \quad , \tag{4}$$

where m_e and m_p denote the masses of the electron and proton, respectively. Similarly, the reduced mass for deuterium μ_D (deuteron-electron system) is

$$\mu_D = \frac{m_d m_e}{m_d + m_e} , \tag{5}$$

where m_d is the mass of the deuteron.

Since the reduced masses given by eqs.(4) and (5) have different values, the Rydberg constants for hydrogen and deuterium will be different. Thus, eq.(2) predicts that an n=3 to n=2 transition will produce slightly different wavelengths for different isotopes. This phenomenon is called the *isotope shift* of the atomic spectra, which led Harold Urey to the discovery of deuterium in 1931, and the Nobel Prize in 1934.

The goal of this experiment is to determine the mass of the deuteron from the measured wavelength shifts between corresponding spectral lines of hydrogen and deuterium.

In naturally occurring hydrogen, a small amount of deuterium is present. When this mixture is excited in a discharge tube, it gives rise to bright lines from the more abundant hydrogen and dimmer lines from the less abundant deuterium, slightly shifted in wavelength.

4.2   Apparatus

You will be given a gas discharge tube containing a mixture of hydrogen and deuterium in such proportions as to make the isotope shift more easily detectable. Also available is a high-resolution grating spectrometer and auxiliary recording equipment that will enable you to measure accurately the wavelengths of hydrogen and deuterium emission spectra in the Balmer series. A photo-multiplier tube (PMT) and oscilloscope will be used as the non-recording detection system. In order to record the spectrum, you are provided with a lock-in amplifier, a DC offset/filter amplifier, a Universal Lab Interface (ULI) to digitize the data, and a computer.
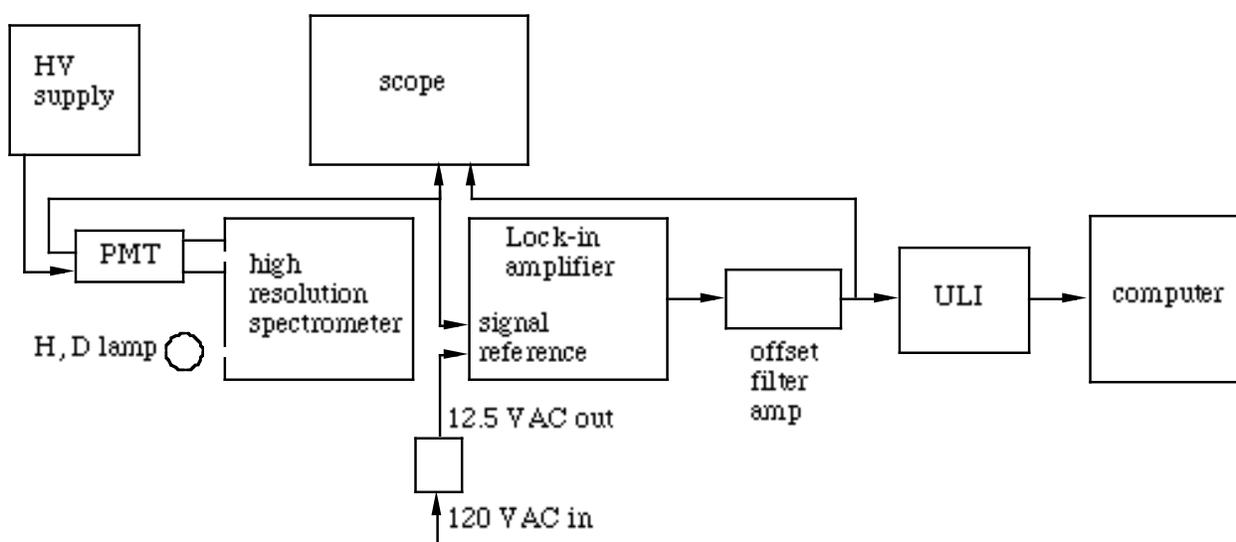

Fig. 3 Mass of the deuteron block diagram

Lock-in amplifier and data recorder.

You may use this apparatus to make a permanent record of the spectral line profiles. The lock-in amplifier is used to extract the small signal from the noise inherent in the spectral lamp. The recommended settings for the Ortec Lock-in amplifier are as follows:

control:	set to:
offset	negative
expand	off

reference	ext 2f
phase	265
time constant	200 ms
sensitivity	100 mV

The spectrometer uses a slowly rotating diffraction grating to select wavelengths. The grating may be driven manually or automatically with a wide range of scanning speeds. The entrance slit width affects the resolution and is adjustable.

The ULI input accepts a 0V to +5V signal and digitizes it with a 12 bit dynamic range (4096 steps). The computer uses Vernier's *Logger Pro* software to collect the data from the ULI and display it on the monitor. You can start the data acquisition software, preconfigured for this experiment, by double clicking on the "*Deuteron Mass DAQ*" icon on the desktop.

There is an amplifier between the output of the lock-in and the input to the ULI. This amplifier can be used to adjust the range of the output from the lock-in so that it falls within the 0V to +5V range required by the ULI.

4.2.1 Set-up

a. Open the entrance slit on the spectrometer to 50μm.

b. Open the PMT slit to the 5th tick mark.

c. Place the hydrogen-deuterium lamp at the entrance slit and tune the spectrometer to the 656 nm line using the λ *in Å* indicator (1nm = 10Å). Which transition does the 656 nm line represent? The spectrometer may be scanned to different wavelengths manually or using the motor-drive as follows:

4.2.2 Manual scanning:

a. Push/pull the HI-LO SHIFT control into its center position. It may be necessary to rotate the SCAN SPEED knob slightly to engage the gears properly.

b. Turn the SCAN SPEED knob to change the wavelength manually. Note that there are four push/pull positions of the SCAN SPEED knob which drive the grating at different speeds. For more precision in reading the wavelength, a divided circle and adjustable pointer have been added to the SCAN SPEED shaft.

4.2.3 Automatic scanning:

Use the three-position λ scan switch (+, off, -) to drive the spectrometer with its motor.

5. Procedure

5.1 Observe the output of the PMT on the oscilloscope as you manually scan across the 656 nm line. It should appear as a double peak. Which peak arises from hydrogen and which from deuterium? How do you know?

5.2 For this pair of Balmer lines only, record a spectrum on the computer. For instructions on how to setup the DAQ and record a spectrum see the section titled **Recording data on the computer,** below. In the neighborhood of this line pair, perform a λ vs. x calibration of the digitized data. Use the wavelength dial of the spectrometer as the wavelength standard. This calibration should enable you to determine the isotope shift, $\Delta\lambda$, to a precision of about 0.01 nm.

6. Data Analysis

Combining eqs.(2), (3), (4) and (5), derive an expression for the mass of the deuteron as a function the measured wavelengths, the principal quantum numbers for the 656 nm transition, and known physical constants.

Using your measured wavelengths, calculate the mass of the deuteron (in kg) together with its uncertainty. Is your result consistent with the notion that the deuteron is a proton and a neutron bound together?

Notice that the peak widths (full width at half maximum) of the hydrogen line differs from that of deuterium. If Doppler broadening contributes significantly to the peak widths, which peaks should appear broader, those of hydrogen or of deuterium? Derive an expression for the ratio of line widths due to Doppler broadening. Are your measured line widths consistent with the Doppler broadening hypothesis?

7.   Recording data on the computer

In order to record a spectrum on the computer you must first adjust the input to the ULI so that it remains in the range 0V to +5V as the spectrometer scans over the spectral lines.

a. Begin by setting the spectrometer on the larger peak of the 486 nm Balmer pair.

b. Start the *Deuteron Mass DAQ* program. Under the *Experiment* menu make sure that *Live Readouts* has a checkmark beside it. You should see the voltage being read by the ULI in a window at the lower right corner of the screen.

c. Monitor the input to the amplifier and the output of the amplifier on channels 1 and 2 on the scope.

d. On the amplifier box, set the *Normal/Inverted* toggle switch to give a + output voltage. Also set the *Time Constant* fully counter-clockwise.

e. Set the spectrometer wavelength on the brightest line. Adjust the amplifier gain to give about +4.7V. While you are adjusting the gain, watch the waveform of the amplifier output on the scope. If the output waveform (channel 2 on the scope) becomes distorted relative to the input waveform (channel 1 on the scope), reduce the gain to obtain the largest undistorted waveform. Depending on the magnitude of the voltage across the amplifier input, it may not be possible to get the output up to 4.7V without distortion.

f. Now set the spectrometer to a region of the spectrum away from the spectral lines where the signal is at a minimum.

g. Adjust the amplifier's *DC Offset*. so that the reading on the computer is slightly above 0V, say ~0.3V. Now go back to the larger spectral line and recheck its reading on the

computer. The gain and DC offset are coupled and adjusting one can change the other. It may be necessary to iterate between the two adjustments a few times to get everything satisfactory.

h. Once the above adjustments are finished, position the spectrometer at a wavelength just below (shorter wavelength) the first line of the pair making sure to approach this wavelength from below so as to eliminate any backlash in the gears that drive the grating.

i. Set the spectrometer drive to scan at the slowest speed.

j. Start the DAQ by either clicking on the *Collect* button, or hitting the [return] key.

k. Once data begins appearing on the graph, start the spectrometer scanning in the positive direction. The DAQ will run for 200 seconds or until you click the *Stop* button or hit [return].

l. When the spectrometer has scanned through both lines in the pair, stop the DAQ.

8. Software configuration.

If you start Logger Pro from the *Deuteron Mass DAQ* icon, the software should be properly configured. The following settings should be already configured in the Logger Pro program:

9. Sensor configuration

a. From the <SETUP> menu select <SENSORS> followed by <SENSOR SETUP>

b. Click on DIN1 and set the sensor to VOLTAGE.

c. Click on the <DETAILS> tab.

d. Click on DIN1 and make sure the Lock/Unlock button is toggled to Lock. Set the information fields as follows:

   i)   LABEL = PMT Signal

   ii)   UNITS = V

   iii) SHORT LABEL = V

   iv)   INTERCEPT = 0

v)   SLOPE = 1

vi)   Click on <OK>.

9.1   Graph Window

a.   Click on the X-axis label and select <Time>.

b.   Set the X-axis limits to [0-200] by clicking on the left and right most tick marks.

c.   Click on the Y-axis label and select <PMT Signal>.

d.   Set the Y-axis limits to [0-5] by clicking on the bottom and top most tick marks.

e.   Configure the Table Window.

9.2   Data Collection Settings

a.   From the <SETUP> menu select <DATA COLLECTION>.

b.   Select the <MODE> tab.

c.   Make sure mode is set to REAL TIME COLLECT.

d.   Select the >SAMPLING> tab. Make sure the following are set.

i.   EXPERIMENT LENGTH = 200 seconds.

ii.   SAMPLING SPEED = 5 samples / second.

iii. Select the <AVERAGING> tab and set averaging to 250 pt.