Polarization Studies of a 3D Photonic Crystal using Transmission and Reflection Experiments

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We present a polarization study of a 3D photonic crystal suitable for an undergraduate advanced physics laboratory or senior project. The crystal is made of 200-nm diameter polystyrene microspheres which forms a face-centered cubic structure and has a pseudo-bandgap in the $[111]$ direction. We probe this bandgap using polarized light in both transmission and reflection geometries as a function of incident angle. Using the Bragg-Snell law, we derive an effective refractive index of 1.40 for each case, which agrees with the effective medium theory. Application of a new analysis based on the condition of equal heights of primary and secondary reflection peaks in s-polarization, and the collapse of p-polarized reflection for large incident angles, also gives the same effective refractive index. These experiments can be performed by students with a basic knowledge of optics such as use of polarizers, lenses, and spectrometers, and the depth of theory covered can be tailored to meet student needs.

I. INTRODUCTION

A photonic crystal (PhC) is a composite material that has a periodic dielectric constant and can be constructed in one, two or three dimensions [1]. Since their discovery in 1987 [2, 3], PhCs have been an active area of research for their fundamental light-matter interactions [4] and engineering applications such as photovoltaics [5] and metamaterials [6]. In addition, PhCs also exist in nature as evidenced by some beetles [7] and the skin of panther chameleons [8].

The periodicity of the PhC is typically on the order of the wavelength of visible light, which means an incident beam can undergo Bragg diffraction. As a result, there is a region of the spectrum called the photonic bandgap for which light is forbidden to propagate through the crystal in certain directions. The wavelengths that are strongly diffracted depend on the effective refractive index (n_{eff}), angle of incidence, and the periodic nature of the PhC. This relationship is quantified by the Bragg-Snell law [4], and can be used to calculate n_{eff} from transmission or reflection data.

Previously, we presented a study on the growth of PhCs made from polystyrene microspheres (artificial opals) and their transmission properties of unpolarized light [9]. The sphere diameters were 200 nm, 220 nm, and 370 nm. They self-assembled into a face-centered cubic (fcc) structure in a water solution to form thin films. Application of the Bragg-Snell law to the spectra revealed that the opals all had the same n_{eff} as expected from the effective medium theory [10].

In this paper, we extend the experiment to include polarized light and introduce a new reflection component. We use the Bragg-Snell law to find n_{eff} from transmission and reflection spectra for s and p polarizations. We also apply a new analysis method to extract n_{eff} based on the criterion of equal

heights of primary and secondary peaks in s-polarized reflection, and the collapse of p-polarized reflection, for large incident angles [11]. To show the complementary nature of both approaches, we present work on a 200-nm opal.

The experiments and analyses described in this paper are suitable for an advanced physics laboratory course or senior capstone project, and builds on the knowledge gained by students in introductory physics courses [12], particularly, those that involve the polarization of light and spectra.

The paper has the following organization. We briefly review the Bragg-Snell law, and then introduce the method proposed in Ref. [11] that allows us to calculate n_{eff} and microsphere size from polarized reflection spectra. After a review of the experiment, we present analyses of the spectra, and discuss the results considering the opal band structure.

II. THEORY

When incident light strikes a PhC, it can be Bragg diffracted if the light wavelength falls in the forbidden region of the spectra corresponding to its photonic bandgap. The diffracted light will appear as a reflection peak or a transmission valley in the collected spectra, and its location depends on the incident angle, lattice spacing, and dielectric constants making up the PhC. The relationship is described by the Bragg-Snell law,

$$m\lambda = 2D\sqrt{n_{eff}^2 - \sin^2 \theta}, \quad (1)$$

which is a modification of Bragg's law to account for refraction into the PhC [4]. Here, m is the order of diffraction, λ is the free-space wavelength of the diffracted light, D is the interplanar spacing, n_{eff} is the effective refractive index, and θ

is the angle of incidence with respect to the normal. Applying Eq. 1 to reflection or transmission data allows us to calculate n_{eff} . Ref. [4] has a simple derivation of the Bragg-Snell law for the interested reader.

The effective medium approach treats a composite system like the PhC as one continuous medium with a refractive index n_{eff} having scattering centers located at lattice points. It holds true for wavelengths much larger than the particle size. However, in PhCs they are of the same order and this approach is only an approximation. An extension of the Lorentz-Lorenz equation by Aspnes connects n_{eff} to those of the constituents by [10],

$$\frac{n_{eff}^2 - 1}{n_{eff}^2 + 2} = f \frac{n_{ps}^2 - 1}{n_{ps}^2 + 2} + (1 - f) \frac{n_{air}^2 - 1}{n_{air}^2 + 2}, \quad (2)$$

where, $n_{ps} = 1.59$ for the polystyrene microspheres, $n_{air} = 1.00$ for air, and $f = 0.74$ for the volume fraction of the spheres in a fcc structure. An agreement between n_{eff} derived from Eq. 1 and Eq. 2 would lend support to the effective medium approximation.

A different approach for extracting n_{eff} that does not rely on the Bragg-Snell law was proposed by Avoine and coworkers, and specifically developed for artificial opals [11]. They noticed that the primary reflection peak, diffracted from the (111) planes, blue shifts with increasing angle of incidence in accordance with the Bragg-Snell law. However, at larger angles, another peak appears diffracted from the $(\bar{1}\bar{1}1)$ planes and further diffracted by the (200) planes into the specular direction. This double-diffracted secondary peak red shifts with increasing angle. There is an incident angle θ_K where the two peaks have the same height, and the effective refractive index is given by

$$n_{eff} = \sqrt{3} \sin \theta_K. \quad (3)$$

The diameter a of the microspheres is related to the wavelength midway between the two peaks λ_K as

$$a = 3\lambda_K/4n_{eff}. \quad (4)$$

Ref. [11] presents a derivation of Eq. 3 and Eq. 4 by analyzing diffraction in reciprocal space, specifically, in the first Brillouin zone.

III. EXPERIMENT

We designed an experiment to study reflection and transmission of polarized light incident on the opals. The setup shown in Fig. 1 is configured for measuring reflection and can be easily reconfigured to measure transmission. All components prior to the opal were secured in a cage-assembly. Since the cage forces the centers of the optics to line up, optical alignment only involves translation of components along the beam path and is simple to do. The assembly also provides a cost effective, compact and robust design.

White light from a tungsten-halogen lamp (360-2000 nm, Ocean Optics, LS-1) was fiber coupled to the assembly, and then slightly apertured through a 1-mm diameter pinhole before collimating with a 50-mm lens. This last step is important because a diverging beam will compromise the data by introducing an array of incident rays and make the interpretation of spectra difficult. The collimated light was then polarized through a Glan Taylor polarizer (Thorlabs, GT10) that was mounted on a cage rotation stage (Thorlabs, CRM1). The mounted polarizer, pinhole, collimating lens, and fiber coupling were part of the cage assembly, and designed this way to make alignment easy for students.

FIG 1. Experimental setup for measuring polarized reflection spectra of opals. The fiber optic from the light source, aperture, collimating lens, and polarizer are secured in a cage assembly. The collection optics is on a rotatable arm.

We placed the opals on a rotation stage (Thorlabs, RP01) and carefully aligned the incident light so it hit the opal surface on the rotation axis. The reflected light was collected with a small lens attached to an optical fiber that was connected to a USB spectrometer (Ocean Optics, Maya PRO 2000). The collection optics were placed on a movable arm and attached to the base of the holder containing the opal. We measured transmission spectra by placing the collection arm directly behind the opal. The percentage transmission and reflection values were obtained by following the standard protocol of comparing measured spectra to that of a reference spectrum—the opal glass substrate for transmission and a silver mirror (Thorlabs, PF10) for reflection.

IV. DISCUSSION

Figure 2 shows the transmission spectra for the 200 nm opal for (a) s-polarization, and (b) p-polarization as a function of incident angle, while Fig. 3 shows the corresponding reflection spectra. Notice the primary transmission minima and reflection maxima blue-shift for both polarizations in accordance with the Bragg-Snell law. A secondary transmission

minima and reflection maxima also appears for $\theta > 30^\circ$ and red shifts. We apply Eq. 1 to our data by setting $m = 1$ (only observe one order) and plotting λ^2 against $\sin^2 \theta$, where λ is the wavelength for primary transmission minima or reflection maxima. A linear fit to the plots reveals an effective refractive index of 1.40 in each case like our previous work with unpolarized light [9]. This result agrees with the expected value of 1.41 from the Lorentz-Lorenz relationship (Eq. 2) and supports the effective medium approximation for the opal.

We now show how to calculate n_{eff} from polarized reflection spectra using the new analysis developed by Avoine and coworkers. From Fig. 3(a), the angle at which the secondary peak equals the primary peak for s-polarization is $\theta_K = 53.5^\circ$. The spectrum for that angle is highlighted in bold red. The midpoint between the two peaks is at 388 nm. Using Eq. 3 and Eq. 4, we find $n_{\text{eff}} = 1.39$ and $a = 205$ nm, respectively. Both values are in good agreement with n_{eff} derived from the Bragg-Snell law and the stated microsphere size of 200 nm.

Beyond finding n_{eff} using two different approaches and comparing results to the effective medium theory, the primary reflection peaks in Fig. 3 presents some interesting physics. We define the peak height to be the reflection maximum minus a baseline reflection value that is away from the bandgap at large wavelengths (see Fig. 3(a)) and define the peak width as full-width at half-maximum (FWHM). The peak height decreases in magnitude a little for s-polarization but does so dramatically for p-polarization. The peak width for s- and p-polarization start out with similar values for normal incidence but then the width for p-polarization narrows noticeably for larger angles while slight narrowing occurs for s-polarization.

The explanation for this behavior lies in how polarized light couples to the energy bands in the opal. The microspheres self-assemble into a fcc structure and diffraction from the (111) planes produces a bandgap in the [111] direction, which happens to be perpendicular to the opal surface. Four bands are responsible for this bandgap and are designated bands 1, 2, 3, and 4 as shown in Fig. 4. The vertical axis a/λ is an energy scale and the horizontal axis is the internal angle within the opal and scales with the propagation vector $\mathbf{k}$.

Bands 2 and 3 are symmetric with respect to mirror reflection on the diffraction plane (the one that contains the incident and diffracted rays) and bands 1 and 4 are antisymmetric. Since p-polarized light is symmetric under mirror reflection, it can couple only to bands 2 and 3. On the other hand, s-polarized light is antisymmetric and couples only to bands 1 and 4 [13]. The energy gap between bands 2 and 3 lessens with increasing angle of incidence, which means (1) the width of the reflection peak narrows, and (2) less p-polarized light is rejected which reduces the peak height. Bands 1 and 4 do not narrow much at all for larger angles and can be used to explain the s-polarized spectra. Further discussion on this topic in terms of the Brillouin zone and high symmetry directions can be found in Ref. [13].

FIG 2. Transmission spectra for the 200 nm opal for (a) s-polarized and (b) p-polarized light show a blue shift of the minima with increasing angle in accordance with the Bragg-Snell law. The numbers in the legend are in degrees.

FIG 3. Reflection spectra for the 200 nm opal: (a) s-polarization shows that equal heights for primary and secondary peaks occurs at 53.5° , and (b) p-polarization shows a quasi-collapse of the reflection peak at 54.5° . The numbers in the legend are in degrees.

FIG 4. Calculated photonic bandgap along the diffraction plane. The point zero corresponds to normal incidence and an internal angle of 30° is about 44° angle of incidence.

Finally, we note a curious feature in Fig. 3(b). The p-polarized reflection peak reaches its smallest value at 54.5° and is reminiscent of $n_{eff} = \tan(\theta_{Br})$, Brewster's law. Using this angle, we find $n_{eff} = 1.40$ in agreement with our previous measurements. However, Brewster's law should not be applicable here because the collapse of the peak is due to bandgap narrowing and not due to reflectivity at the air-opal interface.

V. CONCLUSION

We presented a detailed optical polarization study of artificial opals. In our experience, students taking an advanced physics laboratory course or completing a senior project are well suited to do this experiment. The physics behind it is an appealing medley of optics and solid-state physics, and the

experiment provides a good introduction to the latter. Students can set up the experiment with relative ease because many of the critical optical components are fixed in a cage assembly and only requires translation along the beam path. They can also switch between transmission and reflection geometries by simply rotating the collection arm. Since the spectral features are broad, collecting data in 4°-5° intervals is sufficient to see clear shifts in spectra, and a high resolution spectrometer is not necessary.

While the transmission experiment only requires rotation of the opal and students can collect data quickly, the reflection experiment is more time consuming because of repositioning of the collection arm for each opal angle. Limiting the experiments to smaller number of angles will save students time but will still allow them to verify the Bragg-Snell law. The method of Avoine only requires measuring the critical angle where the two peaks are the same height. As such, students can collect data around the critical angle and narrow down its value. Another idea is to measure the FWHM of the reflection peaks for both polarizations and overlay the peak edges on the theoretical band diagram shown in Fig. 4. This experiment will expose students to optical alignment of incoherent white light sources, polarizers, and fiber optic data collection. More importantly, it reveals how to model the interaction of polarized light with photonic crystals.

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- [1] J. D. Joannopoulos, R. D. Meade, and J. N. Win, *Photonic Crystals: Molding the Flow of the Light*, 2nd ed. (Princeton University Press, Princeton, 2008), pp 1-5.
 - [2] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
 - [3] S. John, Phys. Rev. Lett. **58**, 2486 (1987).
 - [4] E. Armstrong and C. O'Dwyer, J. Mater. Chem. C, **3**, 6109 (2015).
 - [5] S. John, Nature Materials, **11**, 997 (2012)
 - [6] S. Foteinopoulou, Physica B **407**, 4056 (2012).
 - [7] H. Wang and K. Zhang, Sensors **13**, 4192 (2013).
 - [8] J. Teyssier, S. V. Saenko, D. van der Marel, and M. C. Milinkovitch, Nature Communications **6**, 6368 (2015).
 - [9] G. Mayonado, S. M. Mian, V. Robbiano, and F. Cacialli, 2015 BFY Proceedings [College Park, MD, July 22-24, 2015], edited by M. Eblen-Zayas, E. Behringer, and J. Kozminski.
 - [10] D. E. Aspnes, Am. J. of Phys. **50**, 704 (1982).
 - [11] A. Avoine, P. N Hong, H. Frederich, J. Frigerio, L. Coolen, C. Schwob, P. T. Nga, B. Gallas, and A. Maître, Phys. Rev. B **86**, 165432 (2012).
 - [12] J. Kozminski, N. Beverly, D. Deardorff, R. Dietz, M. Eblen-Zayas, R. Hobbs, H. Lewandowski, S. Lindaas, A. Reagan, R. Tagg, J. Williams, and B. Zwickl, *AAPT Recommendations for the Undergraduate Physics Laboratory Curriculum*, American Association of Physics Teachers, (2014).
 - [13] J. F. Galisteo-Lopez, F. López-Tejeira, S. Rubio, and C. López, and J. Sánchez-Dehesac, Appl. Phys. Lett., **82**, 4068 (2003).