

The Magnetic Pendulum

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Nonlinearity is a profound concept in the study of physical systems. The characteristics of seemingly very simple systems may turn out to be extremely intricate and practically unpredictable due to the nonlinearities associated with them. The study of chaos—that essentially has its basis in nonlinear dynamics—begins with the study of such simple systems. The magnetic pendulum can be one such system.

A pendulum is one of the most simple and affluent systems in terms of its mathematical basis and the range of fields of science that it can relate to. Without a doubt, it is a gift of reflective simplicity for our reductionist science. With slight modifications, it can exhibit even more insightful phenomena, chaos being one of them. In this experiment, we will explore the notion of chaotic dynamics using a “magnetic pendulum”.

KEYWORDS

Determinism · Chaos · Supersensitivity · Phase Portrait · Poincare Map · Attractor · Fractals · Self-similarity · Resonance · Rotary motion sensor.

PREREQUISITE EXPERIMENT: Chasing Chaos with an RL-Diode Circuit

APPROXIMATE PERFORMANCE TIME: 1 week

1 Objectives

In this experiment, we will discover:

1. how apparently simple systems can be highly non-linear and exhibit a complex behavior under certain conditions,
2. how non-linearity can be made more prominent using simple methods and hardware modifications,
3. simplistic richness of the mathematical and physical structure of dynamical systems,

4. the conditions and consequences of the notion of super-sensitivity and its relationship with chaos,
5. how to tell chaos apart from statistical indeterminacy,
6. how to construct and interpret phase portraits and Poincare Maps for different kinds of responses of a system,
7. how fractals are associated with attractors and are manifest in the graphical data of such systems.

References and Essential Reading

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2 Introduction

The Basics: Write the equation of motion for a simple pendulum and identify the terms that can represent the associated nonlinearity.

Linearize: Under what conditions, a pendulum can be treated as a linear system?

Almost all of the known physical systems are essentially nonlinear. Yet, for simplicity, they can be treated as linear systems within some operating constraints. The magnetic pendulum is one such system: although it is treated as a linear device in elementary mechanics, when treated accurately, it is a nonlinear system. So, it can help us to look into some nonlinear phenomena like chaos.

3 Apparatus

The magnetic pendulum capable of exhibiting chaotic dynamics is required to fulfill the following demands:

- variable amplitude and frequency of the driving force,
- variable natural frequency of the oscillator and the extent of nonlinearity,
- adjustable damping force mechanism, and
- a way to display the graphical data obtained from the system (such as time series, phase plots, Fourier spectra etc.)

We built one such apparatus whose schematic is shown in Figure 1.

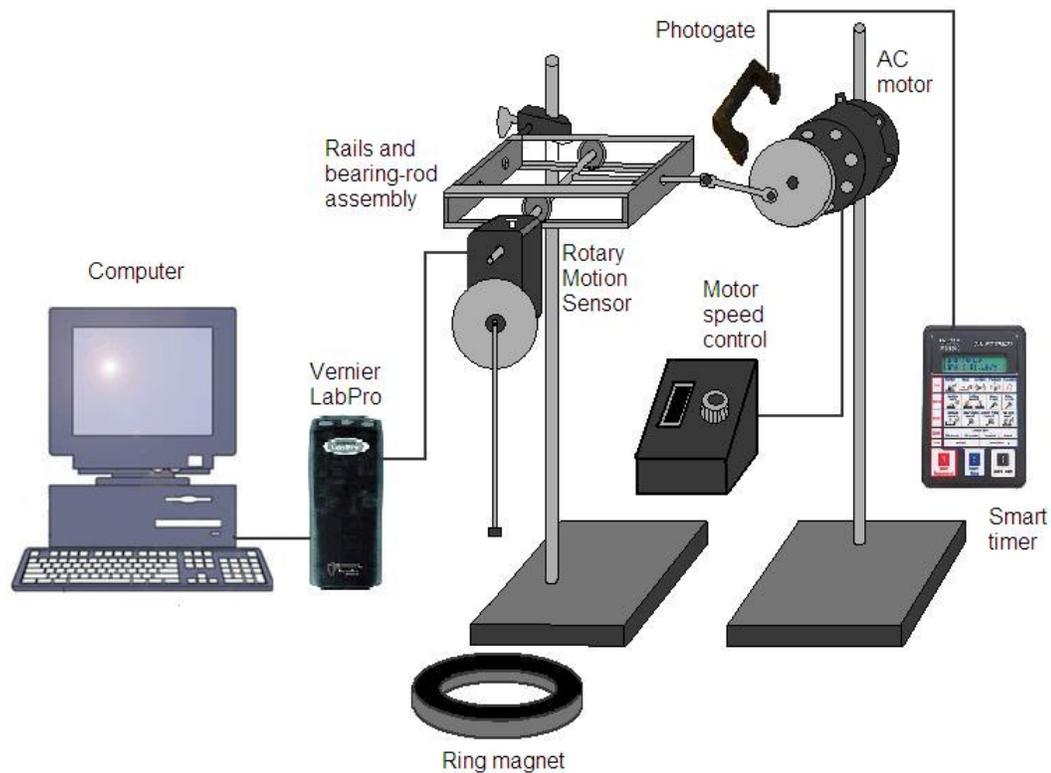


Figure 1: The magnetic pendulum apparatus schematic

The math ingredient: The setup shows a pendulum being driven by a horizontally oscillating pivot. Derive the equation of motion for such a pendulum.

3.1 Setup Description

Some important parts of the setup are briefly described below.

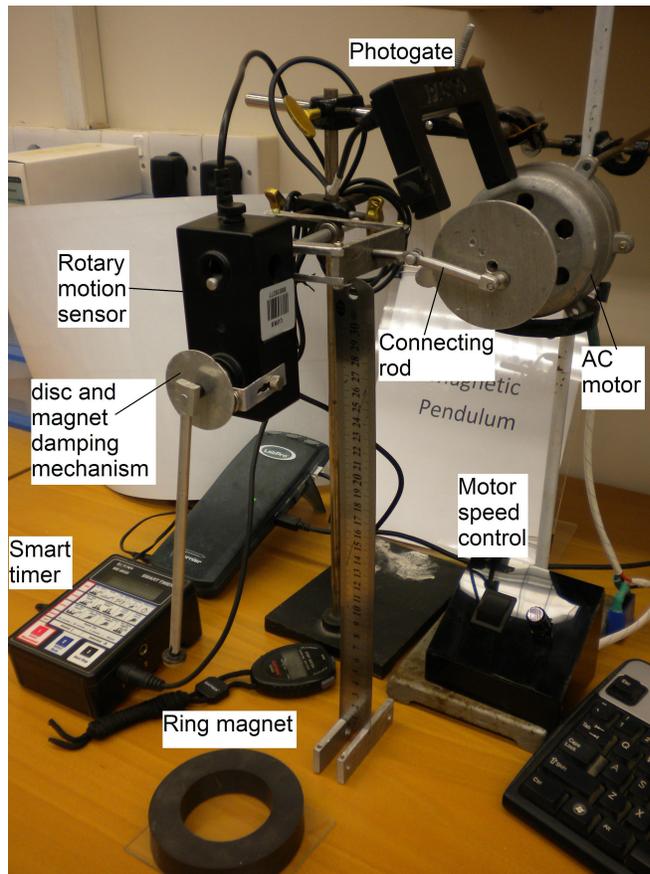


Figure 2: The magnetic pendulum setup

3.1.1 AC Induction Motor

An AC induction motor has been used as the driving device. We use it for its readily available power supply (without any drive needed) and for its ease in speed control, which is simply achieved using a fan-dimmer.

3.1.2 Flywheel and Connecting Rod

The flywheel and connecting rod assembly converts the circular motion of motor's shaft to a linear simple harmonic motion of the bearing-rod assembly to which the pendulum and rotary motion sensor are attached.

Think: What is the advantage of using a flywheel?

3.1.3 Rails and Bearing-Rod-Assembly

A rod with two bearings fitted on it is placed in rails, so as to support the pendulum as well as to keep the rod from undergoing any non-lateral movement.

Ask Yourself: Was it necessary to break the connecting rod into two?

3.1.4 Rotary Motion Sensor

Vernier rotary motion sensor (RMS) encodes the angular information of the shaft into a digital stream and sends it to your LabVIEW program through LabPro. This information can then be used in MATLAB for further processing.

3.1.5 Disc-and-Magnet damping mechanism

A light-weight aluminum disc is integrated with the rod on which the pendulum is suspended. This provides us with magnetic damping (or eddy current damping) when the magnet (attached to the side of RMS) is brought near the disc.

3.1.6 Ring Magnet

Large ring magnet provides a magnetic field to interact with that of the smaller magnet at the end of pendulum. In this way, we can control the magnitude and nature of the restoring torque and hence the nonlinearity of pendulum.

Differentiate: What difference will it make if we use a small magnet similar to the one attached to pendulum instead of large ring magnet?

Design Idea: Can you design a better setup to meet the same qualitative requirements?

4 The Experiment

The experiment is divided into three parts:

1. Exploring the nonlinearities
2. Driving the system into chaos
3. The graphical analysis

4.1 Exploring the Nonlinearities

Let us start our experimental expedition.

1. Measure the time period of pendulum in unforced and undamped mode using a stop watch, keeping the amplitude small.
2. Now, increasing the amplitude, measure the time period again and compare the measurements for several amplitudes.
3. Place the ring magnet under the pendulum and measure the time periods for different amplitudes and ring magnet orientations.

Compare: What can you learn from the measurements of time period, with the ring magnet placed under the pendulum?

Relate: How can you relate the results with the degree of nonlinearity of the system?

4. Run the file **RotaryMotion.vi** from your computer desktop. Collect the data for free oscillations for different scenarios of amplitude and nonlinearity.
5. Import the data into MATLAB and observe the time series, phase plots and Fourier spectra of collected data.

Observe: How is the increased nonlinearity indicated in the graphical data? Can you identify a graph for higher degree of nonlinearity from smaller degrees of nonlinearity?

4.2 Stepping into Chaos

Precaution: Never run the motor at high speeds. It may damage the apparatus. Start from very low speeds and gradually increase when needed. If the motor instantaneously gains speed, switch it off immediately.

1. Place the ring magnet at about a distance of 7cm under the pendulum so that you can clearly observe the stable regions on both sides of the pendulum's mean position.
2. Turn the motor dimmer anticlockwise to its ZERO and switch the motor ON. Now slowly turn the dimmer clockwise until the motor starts.
3. Measure the angular frequency of the drive using the smart timer and make any adjustments if needed. Keep the angular speed in the range of 1 - 2 cycles per second. Measure for both periodic and chaotic behaviors.
4. Once the speed is stabilized, run the LabVIEW file (RotaryMotion.vi) and collect the data for different drive frequencies and different spacings between the ring magnet and pendulum.
5. Import the data into MATLAB and plot the time series, phase plots and Fourier Spectra of the data. Have them saved in a folder named as your roll number and get them printed to attach in the notebooks.

Juxtapose: How are the periodic and chaotic responses different in the graphical representation? How a more nonlinear periodic response differs from a lesser one?

Poincare Sections

6. Drive the system to exhibit periodic response. By putting the sample time equal to the drive frequency, collect the data and plot it as a phase plot.
7. Now, driving the system into chaos, and again setting the sample time equal to the drive cycle, plot the data in the phase space.

Contrast: Observe the difference in the Poincare Maps for the two cases and explain its underlying concept briefly.

4.3 The Graphical Analysis

The purpose of this section is to get acquainted with the graphical richness of chaos and the notion of attractors.

Point Out How graphical data helps in determining if the system is chaotic or not?

Delve deeper: What should a chaotic phase portrait should look like?

Make Out: Can you identify the attractors, repellers or saddle points in the phase portrait, if any?

Magnify: Can you find a connection between attractors and fractals from the graphical data?