# Simple undergraduate lab experiment showing the quantized conductance of nanocontacts

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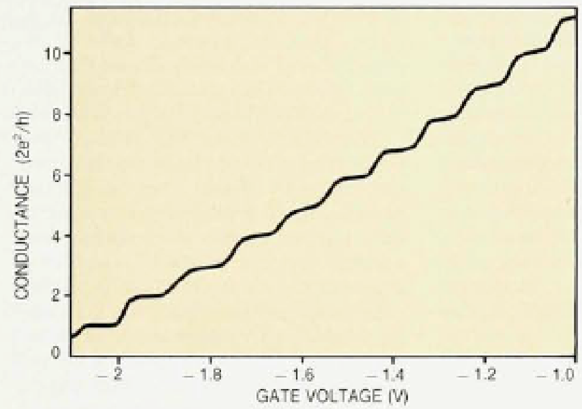
Outline:

- "Quantization" of conductance (=1/resistance) in nano-sized wires was seen ~25 years ago in research labs.
- A few years later, it was discovered that transient nano-wires can be easily formed without any special apparatus.
- We have used this for an junior-level lab experiment, requiring little equipment beyond a digital oscilloscope.
- How to explain conductance quantization using only undergraduate physics.

1988: Two research groups discover that conductance (*I/V*) in nano-size channels is quantized in multiples of $G_0 = 2e^2/h$.

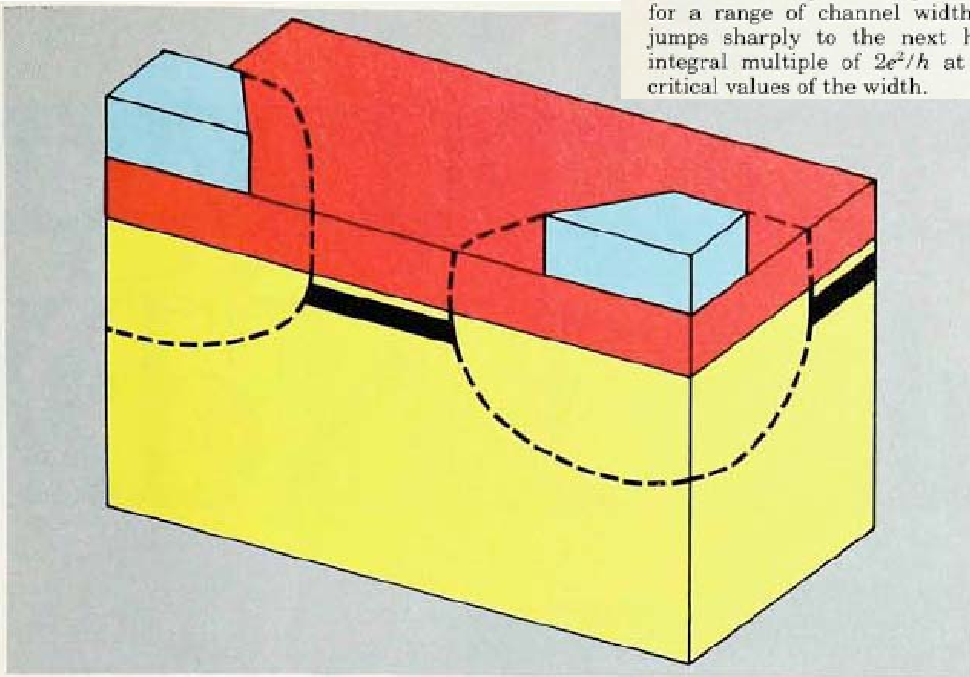
# BALLISTIC ELECTRON TRANSPORT THROUGH A NARROW CHANNEL IS QUANTIZED

The electrical conductance of a short and narrow conducting channel is quantized in units of $2e^2/h$, a team of experimenters from the Netherlands recently reported. The effect was independently observed by experimenters at Cambridge University at much the same time as its discovery by the Dutch group. In the new effect the conductance of the channel, which connects two-dimensional reservoirs of electrons, remains constant at an integral multiple of $2e^2/h$ for a range of channel widths but jumps sharply to the next higher integral multiple of $2e^2/h$ at some critical values of the width.




Conductance of a narrow channel connecting two two-dimensional reservoirs of electrons increases in steps of $2e^2/h$ when the width of the channel increases. The more negative the gate voltage is, the narrower the channel is. (Adapted from reference 1.)





Cross section of a GaAs (yellow) and AlGaAs (orange) heterostructure used in the experiments on quantized conductance. The two-dimensional electron gas, here shown as occupying a region of finite width (black), is at the interface between the GaAs and AlGaAs layers. Two gold electrodes (blue) that act as gates are deposited on the AlGaAs layer using electron-beam lithography. A narrow conducting channel is formed when a negative voltage is applied to the gates depleting free electrons or carriers from the regions enclosed by the dashed lines.

This phenomenon depends directly on the quantum mechanical (wave) nature of electrons in matter.

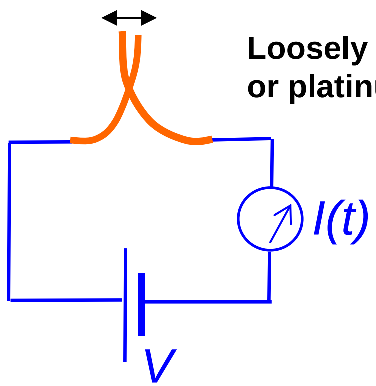
These experiments were carried out at very low temperatures on nanostructures created using semiconductor heterostructures and e-beam lithography – not easy for the teaching lab to reproduce.

## 1995: Costa-Kramer et al. find that loosely-touching wires at room temperature make transient nanowires which show the same conductance quantization

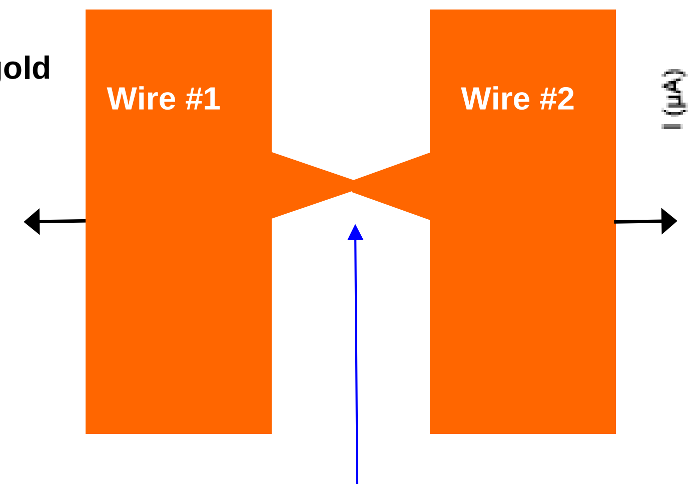
**Surface Science 342, L1144 (1995)**

Nanowire formation in macroscopic metallic contacts: quantum mechanical conductance tapping a table top

J.L. Costa-Krämer [a], N. García [a], P. García-Mochales [a], P.A. Serena [b]




**Loosely touching gold or platinum wires**


**"Bubble gum" model: "necks" might be as small as a single atom!**

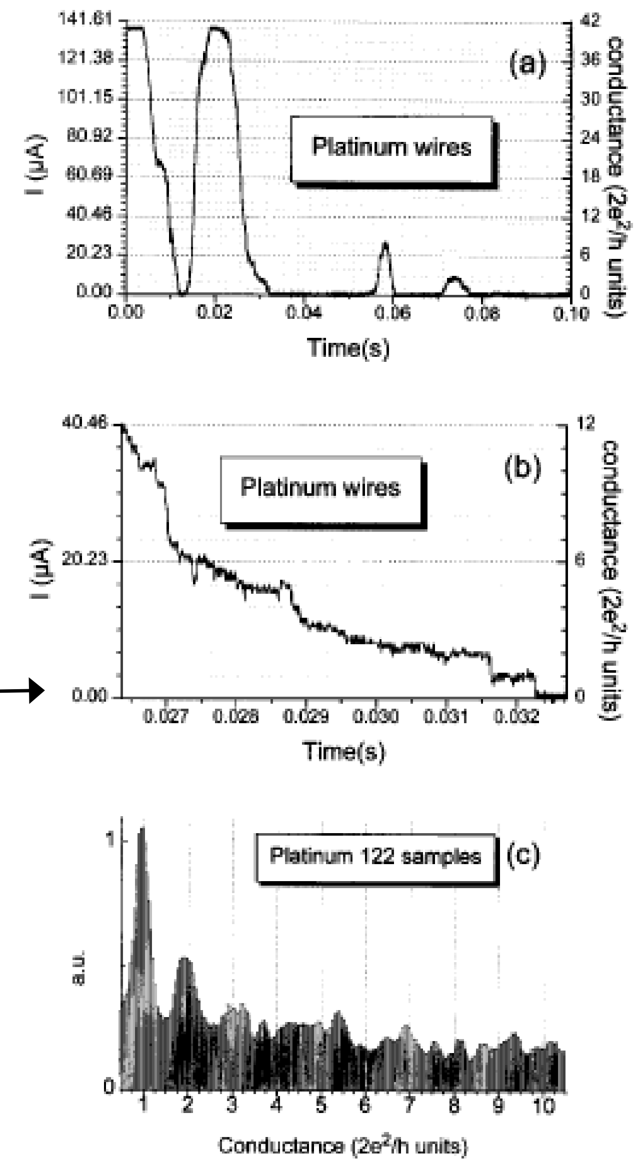



Fig. 2. Conductance experiment for two platinum wires, 0.25 mm diameter. (a) Full current evolution when the wires vibrate around the equilibrium (slightly separated wires) position. Notice evidence of the decreasing vibrational amplitude as less pronounced excursions to the full contact (resistance < 310 Ω for 44 mV voltage difference) position. (b) Detail of the last stages of a contact breakage. (c) Histogram of conductance values made with 122 different curves (a.u. = arbitrary units).

## "Conductance is quantized in units of $G_0 = 2e^2/h$"

Physics questions for your students to think about:

(a) Is the quantum of conductance a tiny compared to everyday quantities, like the quantum of charge $e = 1.6\times10^{19}$ C, or the energy of a photon?

Answer: No, $G_0 = 2e^2/h = 1/13$ kΩ - a very ordinary, every-day value of resistance.

(b) Are observed conductances always precise integer multiples of $G_0$ (as observed charges are always precise integer multiples of $e$)?

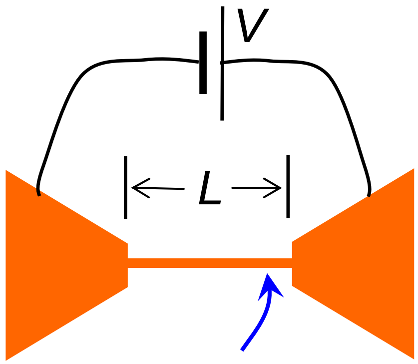
Answer: No, it is easy to find a resistor with $R \neq 1/nG_0$

"Quantization of conductance" is not very helpful terminology. I prefer:

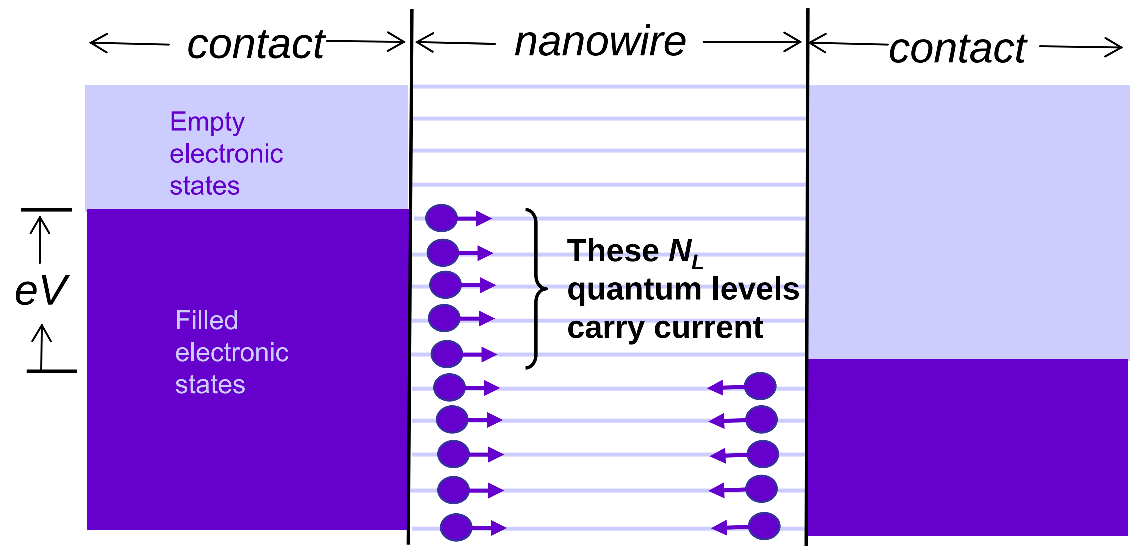
An ideal, one-dimensional wire with no scattering always has the same conductance, $I/V = G_0$.

Very tiny, clean wires act like this. Bigger wires act like several 1D wires in parallel.

## Conductance quantum $G_0 = 2e^2/h$ can be derived using intro-level modern physics (i.e. not assuming a course in solid-state physics)




Quasi-1D nanowire of length $L$, with $N$ mobile electrons moving at velocity $v$




Current in nanowire

$$I = Nev/L.$$

Quantum levels (1D particle in a box):

$$\psi(x) \sim e^{ikx}, \quad k = (2\pi/L)n, \quad n = 1,2,3\ldots$$

$$v = hk/2\pi m = (h/mL)n.$$

$$E = mv^2/2$$

$$\Delta E = eV = mv\,\Delta v = mv\,(h/mL)N_L.$$

$$N_L = eVL/hv$$

Since electron has two spin states,

$$N = 2N_L = 2eVL/hv$$

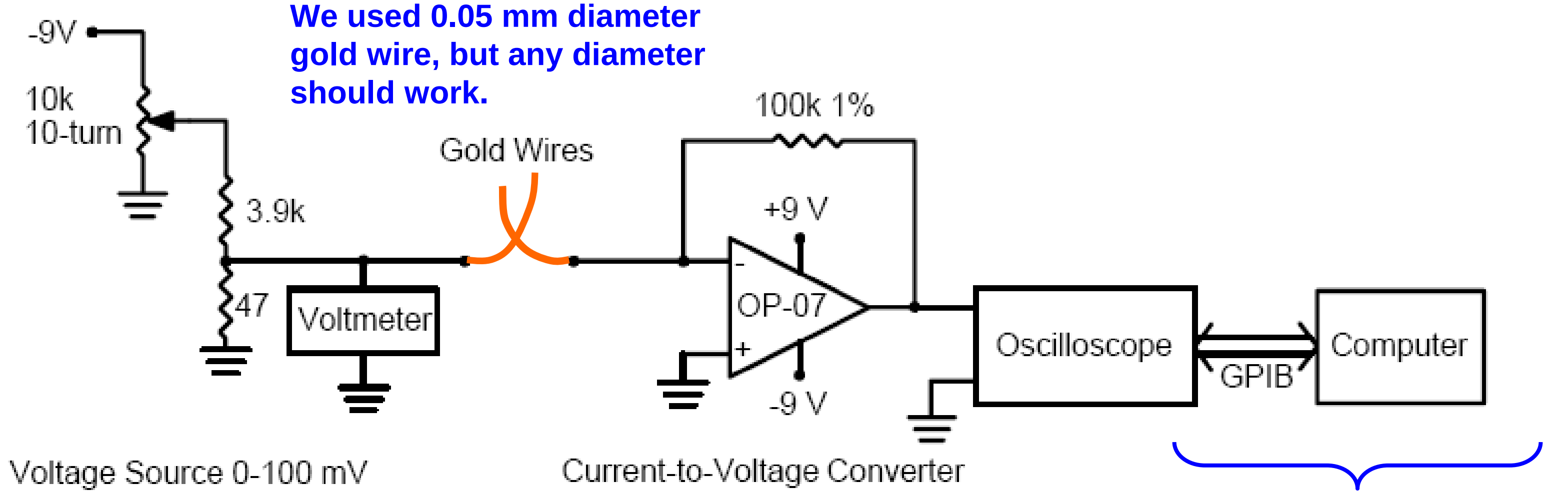
$$I = 2e^2V/h$$

$$\mathbf{G = I/V = 2e^2/h}$$

**Ref: Foley et al., AJP 67, 389 (1999).**

# The apparatus can be very simple and inexpensive (but you need a digital scope or data acquisition board)

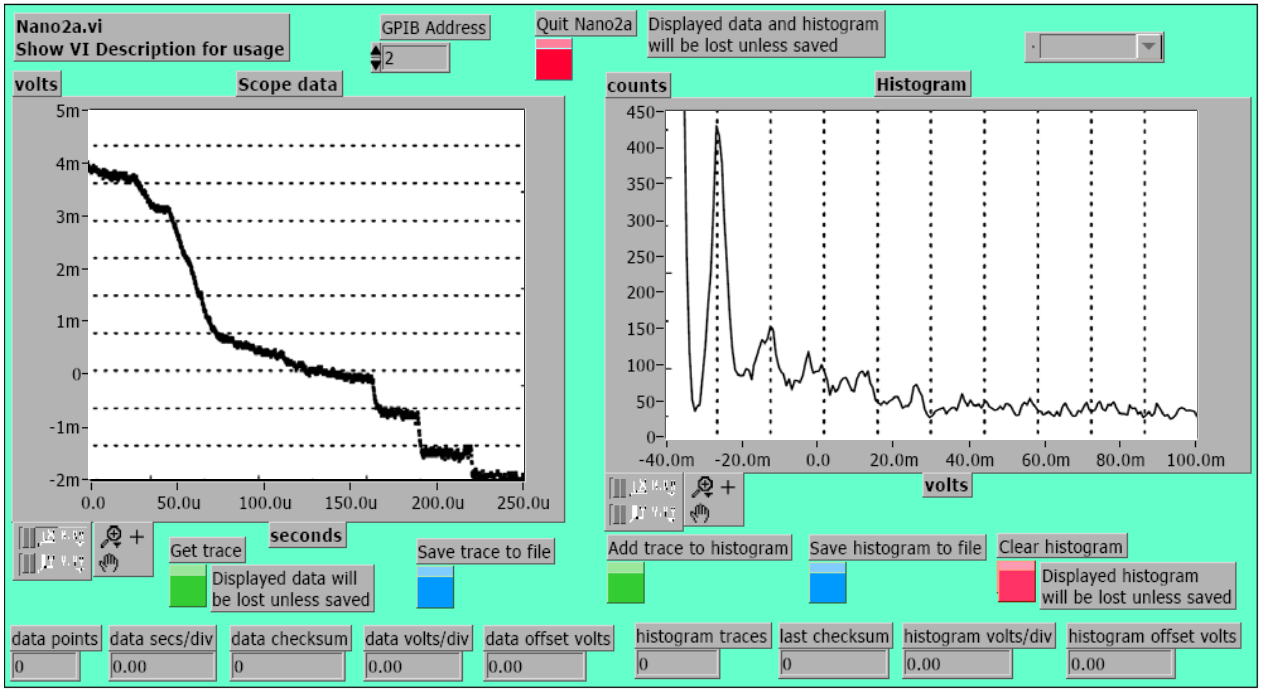
**We used 0.05 mm diameter gold wire, but any diameter should work.**




**Simpler: Can make fixed voltage source ~10mV with 39kΩ and 47Ω resistors.**

**Don't need GPIB or permanently connected PC; can use USB cable or USB stick.**

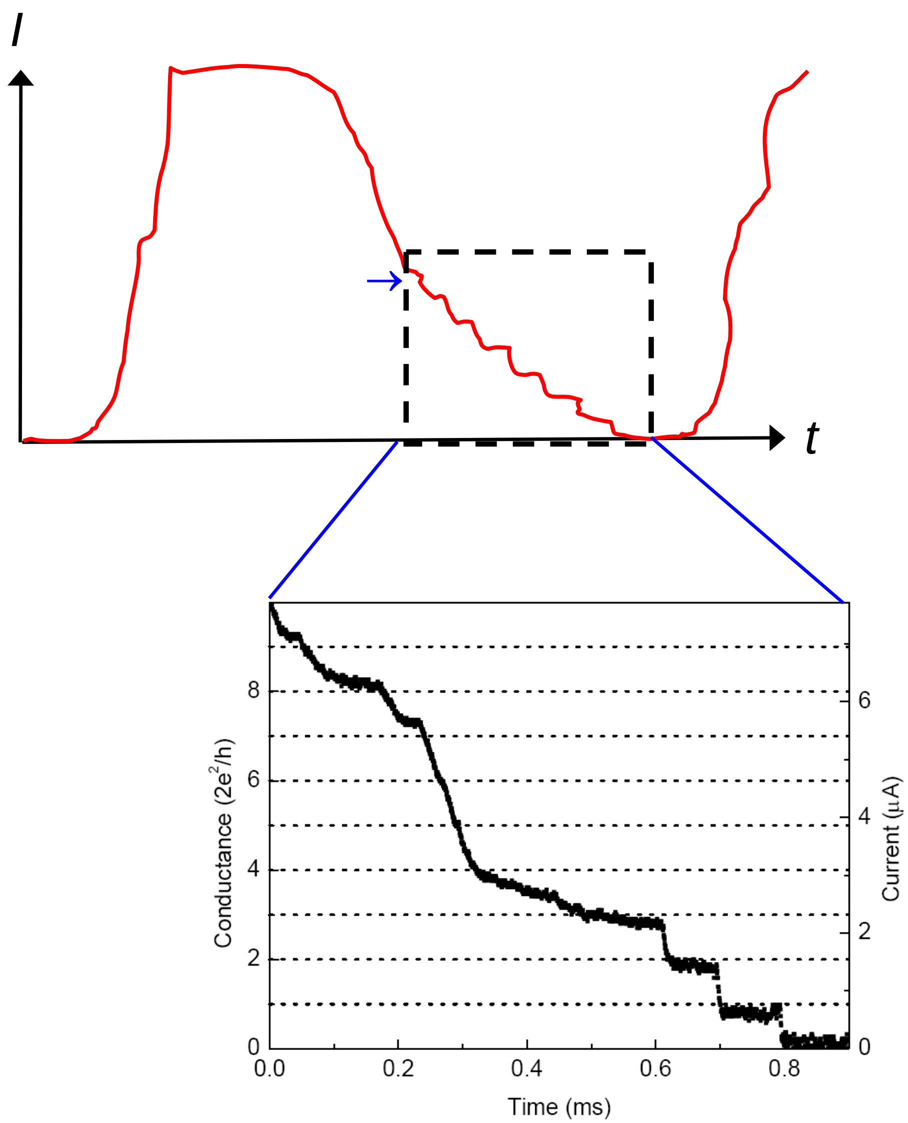
# We wrote a Labview program to read out the scope and show histograms, but this is not necessary




**Can use a USB stick to transfer data to any PC, and any software to analyze and plot the data (Excel, Origin, Igor, Mathcad, etc.)**

# Experimental tips

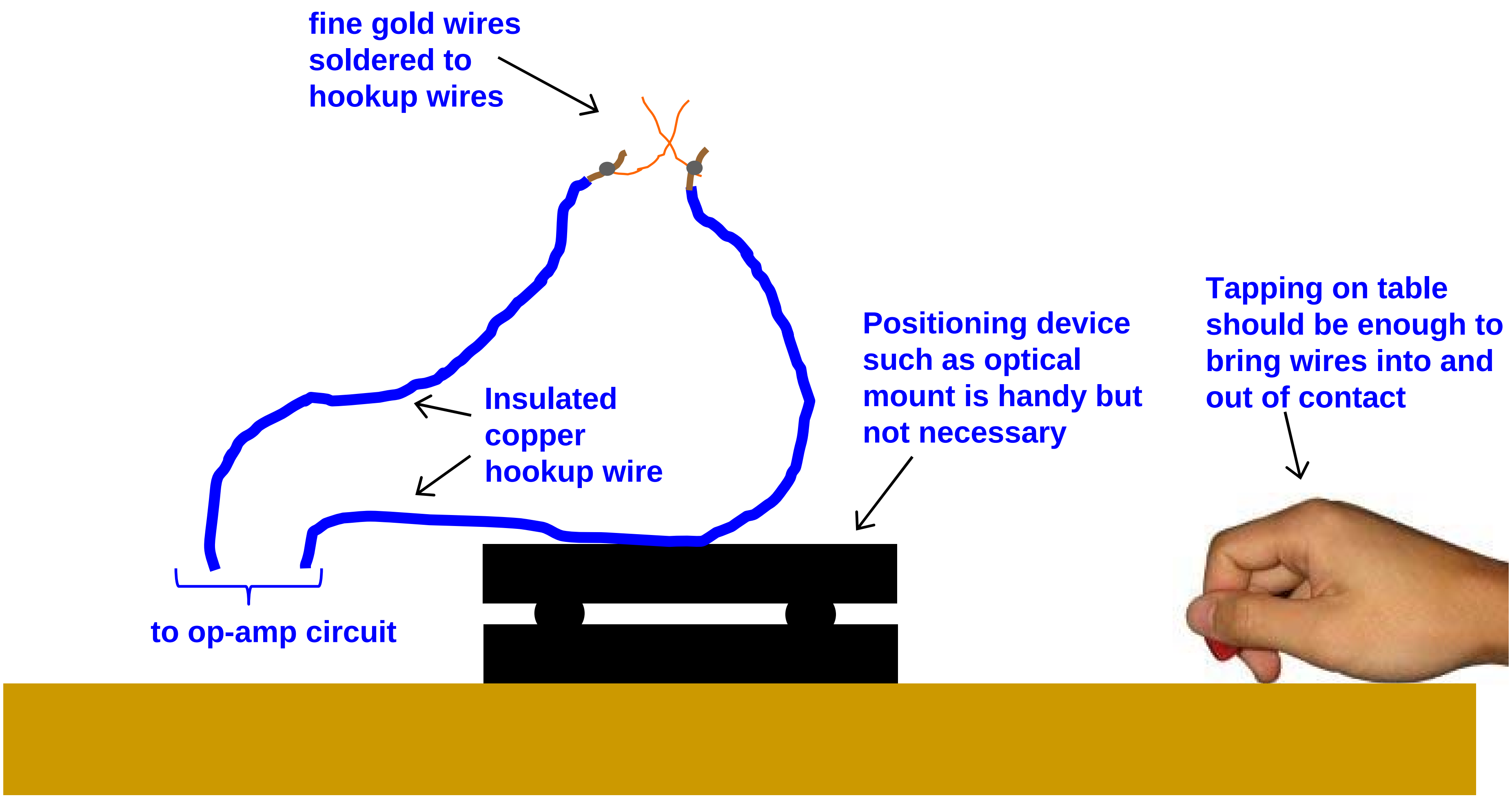
## Setup and triggering of scope is crucial (and mysterious to most students)




**Trigger (→) must be DC, near top of screen, negative slope, auto-trigger off.**

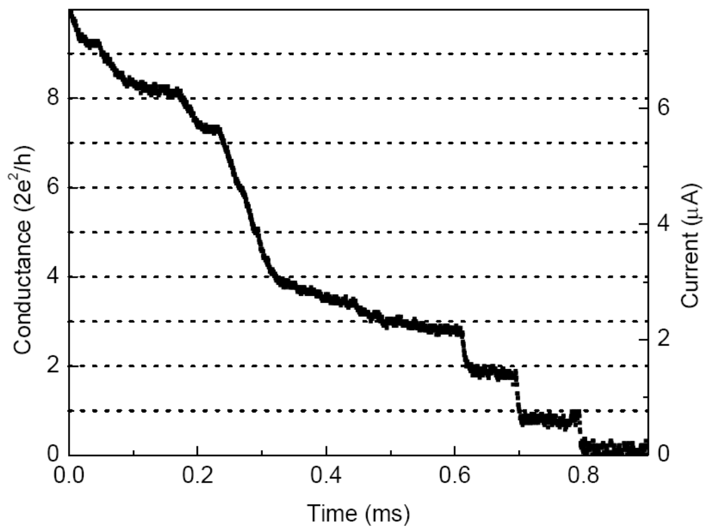
**Time scale roughly as shown (100 μs/div) seems to work best**

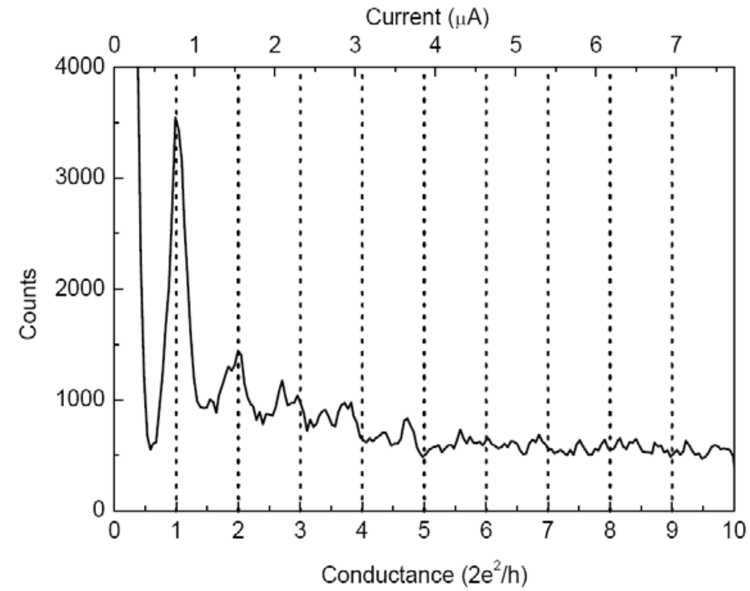
# What the setup looks like



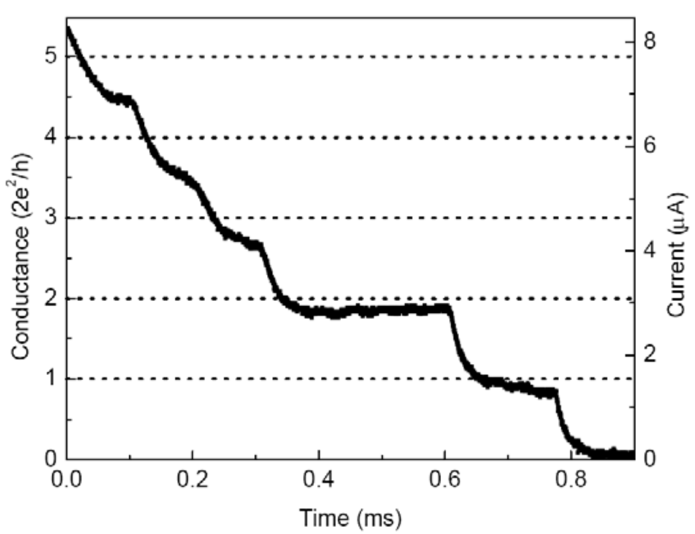
# Data from our junior-level lab course (50 μm gold wires)

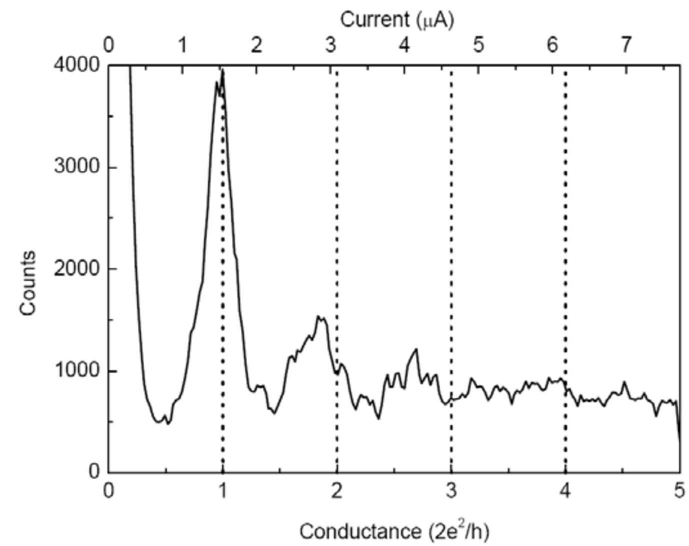
*V* = 10 mV



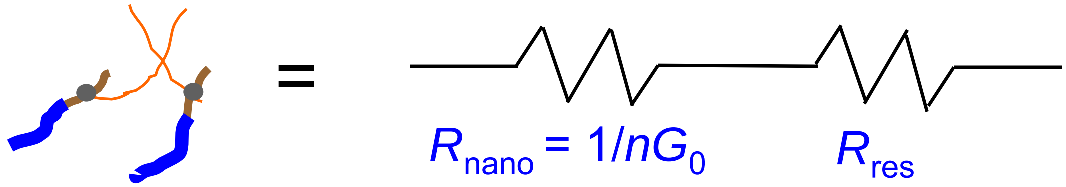




*V* = 20 mV

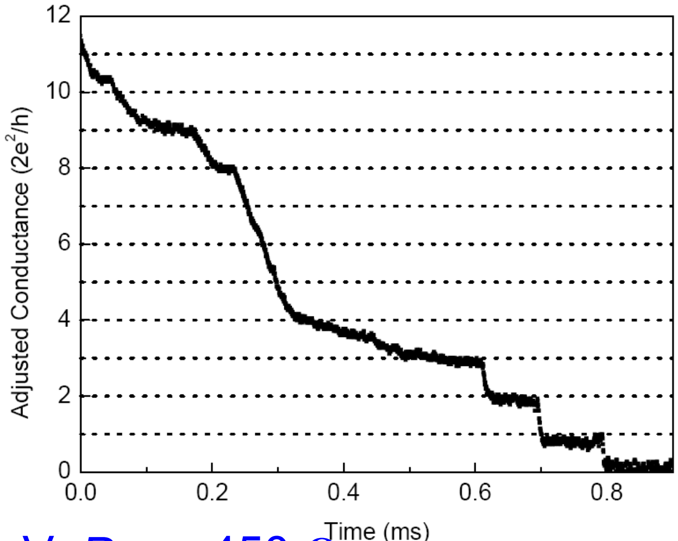


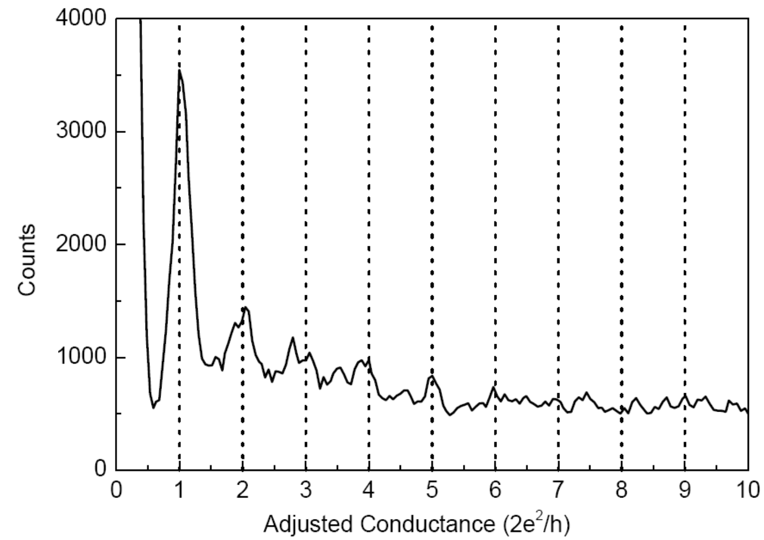



# One way to analyze the data further: “residual resistance” model

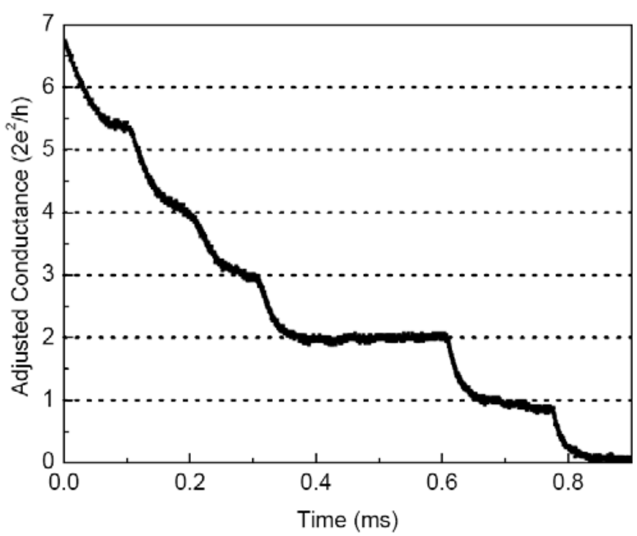


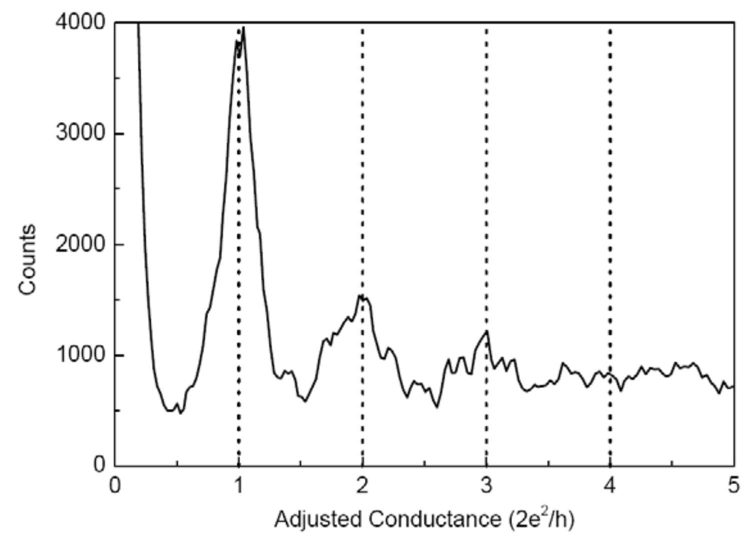

$V$ = 10 mV, $R_{res}$ = 100 Ω




$V$ = 20 mV, $R_{res}$ = 450 Ω



## More sophisticated setups make the data-taking easier (but also require more apparatus)

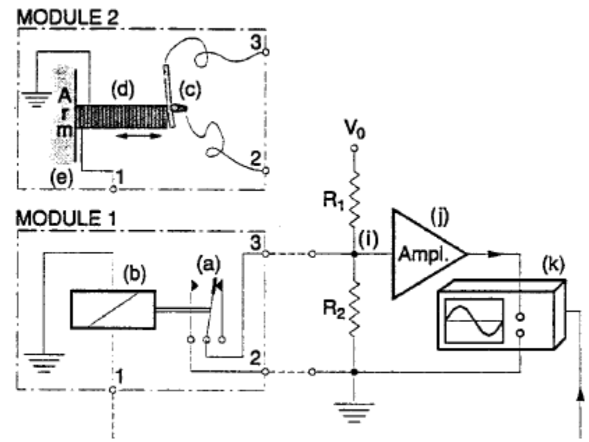


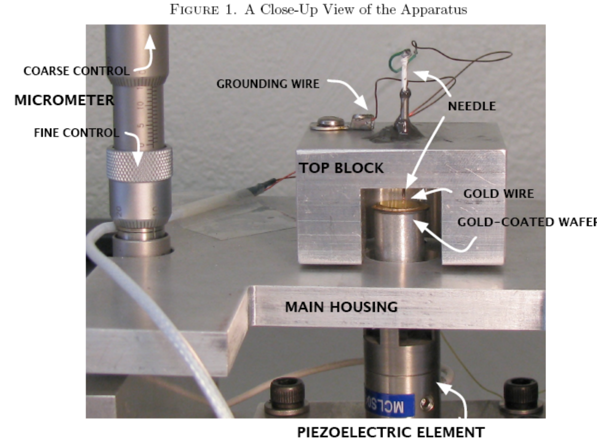

FIGURE 1. A Close-Up View of the Apparatus

### (a) Electromagnetic relay

Hansen, et al., Phys. Rev. B **56**, 208 (1997)

### (b) Piezoelectric positioner

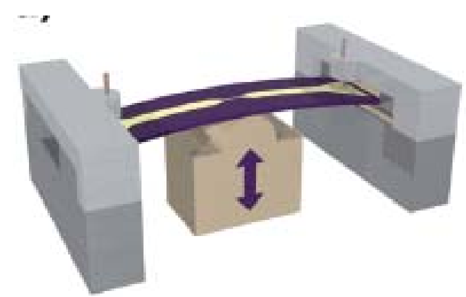
Jeremy Dodd and David Tam, Columbia U.




### (c) Mechanical "break junction"

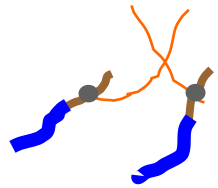
Huisman et al., AJP **79**, 856 (2011).

Tolley et al. talk AG04, this conference.

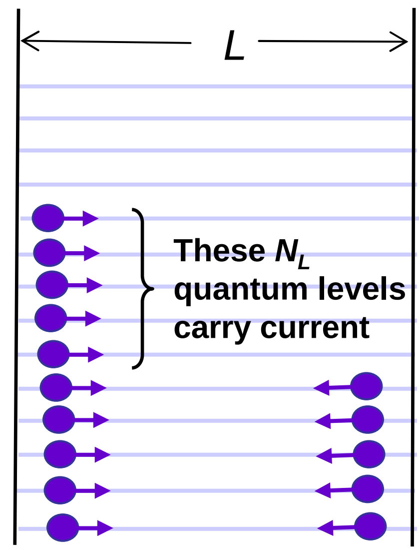


## Summary

• It is easy and cheap to set up an experiment in the teaching lab showing quantization of conductance.



• The physics of quantized conductance is a direct demonstration of the wave nature of electrons in matter, and it can be derived using sophomore-level introductory modern physics (quantum particle in a box + Pauli exclusion principle).




$$\psi(x) \sim e^{ikx},$$
$$k = (2\pi/L)n,$$
$$n = 1,2,3\ldots$$