Uncertainties in Measurements

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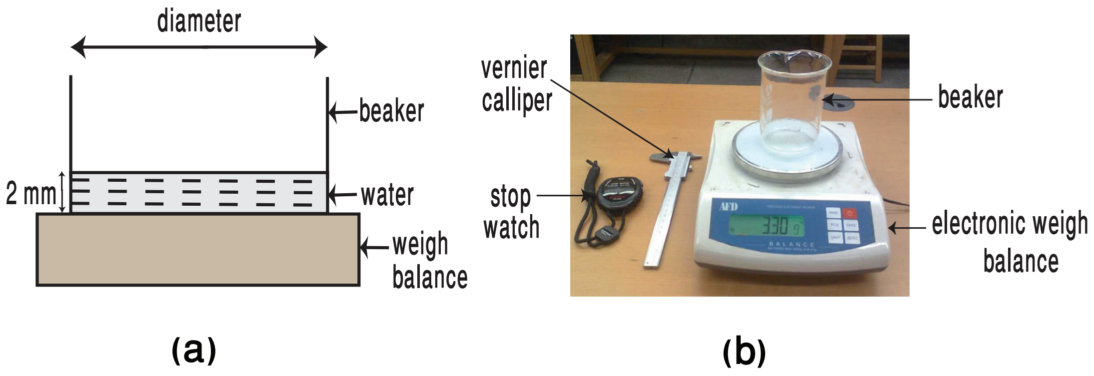
This is an outline of some very simple experiments whose sole purpose is to get familiarized with the concept of uncertainty and how it can be quantified and expressed.

1 Rate of evaporation of water

1.1 Experimental method

Take a small beaker, measure its diameter using the provided vernier callipers. Repeat the measurements for six values of the diameter. Then add water into a beaker, such that the level of the water is around 2 mm from the bottom. Place the beaker on an electronic weigh balance. Use a stop watch to measure the mass of the water at intervals of 300 s. Continue taking readings for approximately 3600 s. Measure and record the room temperature.



(a) (b)

Figure 1: (a) Schematic diagram of the apparatus for measuring the rate of evaporation of water, and its (b) photograph.

1.2 Objectives

1. The best estimate of the evaporation rate per unit area, e is expressed as,

   $$e = \frac{b}{A}, \tag{1}$$

   where b is the evaporation rate and A is the area of the surface of the water exposed to the atmosphere.

2. Express area of the surface in terms of diameter and modify the above expression 1.

3. Calculate the best estimated value of diameter, $D = d + Z$, where d is the mean value and Z is the correction term (due to resolution or the zero error of the device). Evaluate the standard uncertainty in the mean value $u(d)$ as well as in the correction term $u(Z)$. Find the combined uncertainty in D.

4. Evaluate the effective number of degrees of freedom, ν_{eff}, for the combined standard uncertainty in the diameter.

5. Plot mass versus time to find the best estimate of the evaporation rate (after least-squares fitting of your data) and find the standard uncertainty in this value.

6. Calculate the value of e from the modified equation 1 and the uncertainty $u(e)$ by combining uncertainties of diameter and evaporation rate.

7. Find out the effective degrees of freedom for the combined standard uncertainty in the evaporation rate and find the coverage factor at the 95% level of confidence.

8. Calculate the true value of the evaporation rate at the 95% level of confidence.

2 Craters in sand

2.1 Experimental method

Setup the apparatus as shown in Figure (2). Find the masses of the available steel ball bearings. Level up the sand by shaking the container vigorously. Drop the balls, one by one, into the sand container from a range of different heights between 25 cm to 95 cm. The heights are measured from the surface of the sand. Find the diameter D of the crater using a ruler, as shown in Figure (3). For each ball, take at least 3 replicate measurements of diameter at varying heights, so that we can find the mean and spread of the data for a specific height, mass and kinetic energy of the ball. Shake the container vigorously after taking every reading. This is to ensure that the sand is not compacted. Repeat the above process for different balls to get a range of kinetic energies of the impacting object.

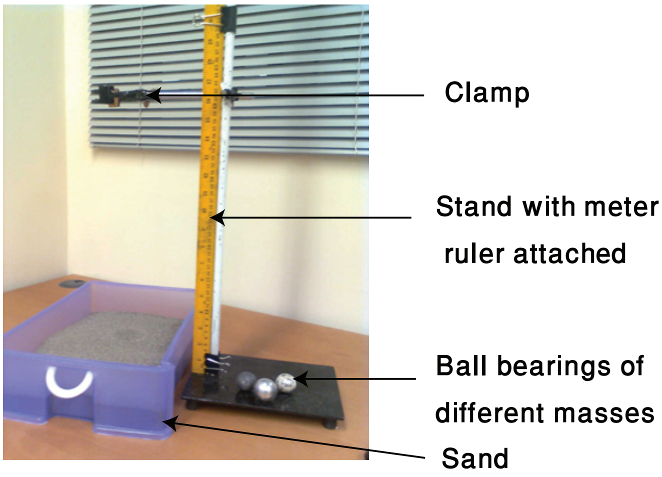


Figure 2: Photograph of the crater formation experiment.

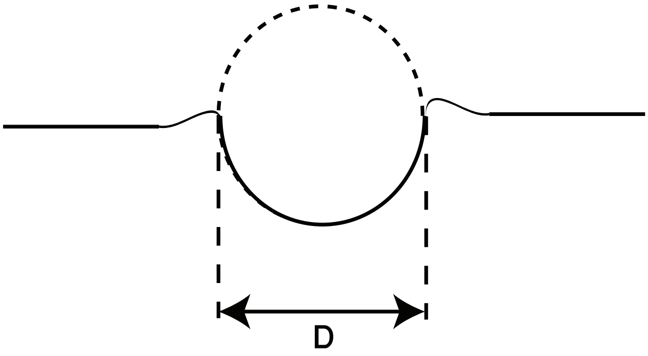


Figure 3: Cross section of the sand crater formed by falling ball into the sand container. The position of the peak of the crater wall is used to estimate its diameter.

2.2 Objectives

1. The diameter of the crater, D, and the kinetic energy, E, of the impacting ball are related through the relation $D = cE^n$, where E is the kinetic energy calculated by assuming that all the potential energy possessed by a ball at height h is transformed into kinetic energy before impact.

2. Determine the best value of and the combined standard uncertainty in the kinetic energy, E, taking into account the uncertainties in the mass, acceleration due to gravity and the height of the ball.

3. Fit the equation $D = cE^n$ to your data using the technique of least-squares. In applying least-squares we assume that the error is confined to the dependent variable. To verify this point compare the percentage uncertainties $\frac{u(x)}{x} \times 100\%$ in the predictor and dependent variables.

4. From the least-squares fitting of your data, find the slope and intercept as well as the standard uncertainties in these values.

5. Your goal is to find n which will specify the mechanism of crater formation [1]. For example, $n = 1/3$ implies that the dominant mechanism is the plastic deformation of the sand surface and $n = 1/4$ suggests that the craters are formed by the ejection of sand.
6. Identify the number of independent degrees of freedom of your data and the coverage factor k at 95% level of confidence.
7. Evaluate the expanded uncertainty in n and state the result in the final form.

3 Coefficient of static friction

3.1 Experimental method

Place a glass slide on a glass block. Move the glass block slowly at an angle of inclination, until critical angle is reached and the glass slide began to slip as shown in Figure (4). Use a protractor of smallest division of 1° to measure the critical angle. Return back the glass block and glass slide to their original positions and repeat the procedure at least for six replicate values of critical angle. Once again, repeat the above steps for a wood piece placed on a glass block.

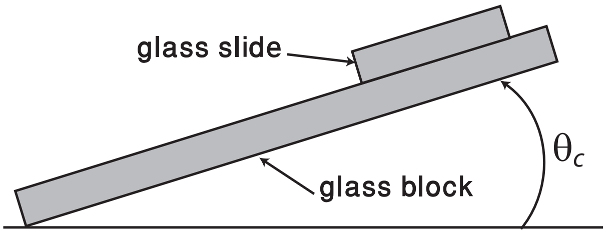


Figure 4: A glass slides on an inclined block of glass. θ_c is the critical angle.

3.2 Objectives

1. Calculate the best estimated value of the critical angle, $\theta_c = X + Z$, where X is the mean of values obtained through repeated measurements and Z accounts for the effect of systematic error (least count or zero error etc.).
2. Find the standard uncertainty in the values of X and Z and then the combined uncertainty in θ_c.
3. Evaluate the effective number of degrees of freedom for the combined standard uncertainty in the critical angle, ν_{eff}, using Welch-Satterthwaite formula.
4. Our aim is to find out the coefficient of static friction, for which we will use the equation,

$$\mu_s = \tan(\theta_c). \tag{2}$$

(This relationship is derived from the formula, $F_{\mathrm{s,max}} = \mu_s N$ where $F_{\mathrm{s,max}}$ is the force of static friction and N is the normal force exerted by the inclined plane on the sliding object).

5. Calculate the best estimated value of the coefficient of static friction (first for glass on glass and then for wood on glass).

6. Now evaluate the standard uncertainty in μ_s.

7. Find out the coverage factor from the calculated effective number of degrees of freedom and determine the expanded uncertainty for the true value of the coefficient of static friction at the 95% level of confidence.

4 Density of a metal

4.1 Experimental method

Place the metal ball bearing on an electronic weight balance of resolution 0.01 g. Note down the mass of the ball and repeat the measurements for a set of eight readings. Measure the diameter of the steel ball bearing using a screw gauge of resolution 0.01 mm. Take a set of six replicate measurements.

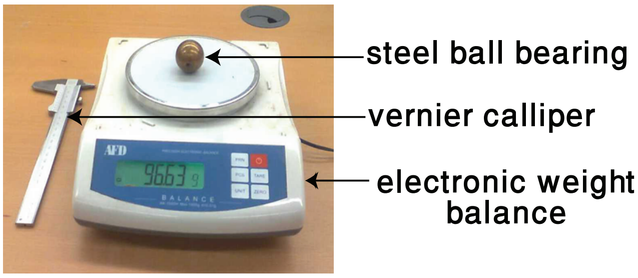


Figure 5: Measuring the mass of metal balls.

4.2 Objectives

1. Calculate the best estimated value of mass, $M = m + Z$, where m is the mean value and Z is the correction term (correction due to resolution or the zero error). Evaluate the standard uncertainty in the mean value as well as in the correction term. Find the combined uncertainty in M.

2. Repeat the above step for diameter's best estimate and its standard uncertainty by combining both A and B type uncertainties.

3. Evaluate the effective number of degrees of freedom, ν_{eff}, for the combined standard uncertainty in mass using the Welch-Satterthwaite formula.

4. Now evaluate the effective number of degrees of freedom, ν_{eff}, for the combined standard uncertainty in dianeter using the Welch-Satterthwaite formula.

5. Find the best estimate for the density of the ball.

6. Now using the standard uncertainties in mass and diameter, calculate the combined uncertainty in the density.

7. Find the effective number of degrees of freedom for the density. Evaluate the coverage factor at the 95% level of confidence.

8. Determine the true value of the density by evaluating the expanded uncertainty at the 95% level of confidence.

References

[1] J.C. Amato, R.E. Williams, Crater formation in the laboratory: an introductory experiment in error analysis, Amer. J. Phys. **66**, 141 (1998).

[2] L. Kirkup, R. B. Frenkel, "*An Introduction to Uncertainty in Measurement*", Cambridge University Press, (2006).