

# Primer on the Lock-in Amplifier

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The resistance of a Cu wire is so small that it cannot be accurately and reliably measured by ordinary voltmeter. The same is true for a gold nanowire tethered between two supports. In experiments, we repeatedly come across situations where we want to measure the response of a system and the response is feeble or is buried in excessive noise. Through this experiment we introduce the art of precise measurements using a lock-in amplifier which is an integral component of several experiments in the physics lab. As such, the present experiment is a pre-requisite to some of the advanced experiments in this course.

## KEYWORDS

Lock-in amplifier · White noise · Flicker noise · Thermal noise · Phase sensitive detection · Low-pass filter · Optical chopper

**SCHEDULED TIME** 1 WEEK

## 1 List of Equipment

1. Optical Chopper, SR540, Stanford Research Systems (SRS)
2. HeNe Laser with Mount, HR020, Thor Labs
3. Lock-in Amplifier, SR510, SRS
4. Photodetector, 818-SL, Newport
5. Polarizers, LPVIS050, Thor Labs
6. Oscilloscope
7. Signal Generator
8. Optical Breadboard
9. Electrical Breadboard
10. BNC cables
11. Connecting Wires
12. Resistors

## 2 Objectives

In this experiment, we will,

1. eliminate unwanted noise, extract signals buried in excessive noise using lock-in amplifier.
2. learn about different types of noise, and see how noise can obscure the required signal and how phase-sensitive techniques can be used to recover the signal.
3. operate optical chopper to measure weak optical signals modulated at a certain frequency.
4. find the minute resistance of a conducting wire.

## References and Essential Reading

- [1] K.G. Libbercht, E.D. Black and C.M. Hirata, "A basic lock-in amplifier experiment for the undergraduate laboratory", Amer. J. Phys. **71**, 1208-1213 (2003).
- [2] Paul Horowitz, W. Hill *The Art of Electronics, 2nd Edition* {Cambridge}, Pg. 184,430-433.
- [3] Manual, Lock-in Amplifier, Model SR510, Stanford Research Systems.
- [4] Manual, Optical Chopper, Model SR540, Stanford Research Systems.
- [5] Manual, Photodiode detector, Model 818-SL, Newport.

## 3 Theoretical Introduction

### 3.1 Noise and its types

Noise include all those voltages and currents that accompany and obscure the signal of interest. Noise is generally classified into two kinds: white noise and pink noise [2].

White noise has all frequency components, ranging from zero frequency (DC) to infinite frequencies. The graph between the noise power density versus frequency would be a constant line. Thermal or Johnson noise is a type of white noise. It arises from the thermal fluctuations in a resistor at finite temperature. The rms amplitude of thermal noise voltage is:

$$V_{rms,thermal} = \sqrt{4k_B T R B},$$

where  $R$  is the resistance and  $B$  is the bandwidth of measurement,  $T$  is the absolute temperature, and  $k_B$  is the Boltzmann Constant.

Shot noise is another type of white noise which arises due to fluctuation of single charge carriers, e.g. electron flowing through a single electron transistor or a highly attenuated beam of practically single photons impinging on a photodetector.

Pink noise has a power spectral density that decreases with frequency. It has a frequency dependence of  $1/f$ . It is sometimes also referred to as flicker noise.

"Intrinsic" noise sources include resistors, vacuum diodes, p-n junctions etc. In addition to the intrinsic noise sources, there are a variety of "non-essential" noise sources, i.e., those noise sources which can be minimized with good laboratory practices. Some of these extrinsic noise sources are highlighted below.

**Capacitive Coupling** A voltage on a nearby piece of apparatus can couple to a detector via stray capacitance. Although the stray capacitance maybe small, but when coupled in, the resultant noise may be larger than a weak experimental signal.

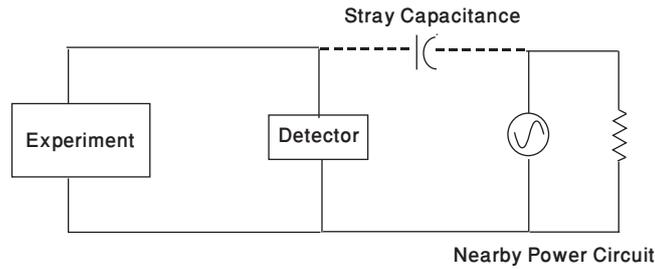


Figure 1: Capacitive Noise Coupling.

It can be cured by removing or turning off the interfering noise source, or by installing capacitive shielding which can be done by placing the setup in a metal box. Consider Figure 1.

**Inductive Coupling** Extrinsic noise can also couple to the experiment via a magnetic field. A changing current in a nearby circuit gives rise to a changing magnetic field which induces an emf in the loop connecting the detector to the experiment. See Figure 2. This can be cured by using a magnetic shielding, or by using twisted pair cables. An example of inductive coupling is the noise due to 50 Hz main power line, called “line interference” or 50 Hz hum. It can be reduced by placing a notch filter which is centered around the line frequency.

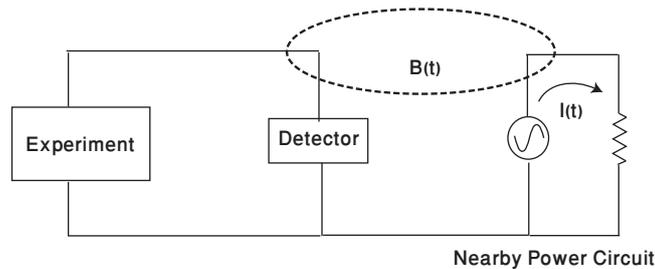


Figure 2: Inductive Noise Coupling.

**Resistive Coupling** Currents through common ground connections can give rise to noise voltages. This arises when we have two different grounding points which are not at exactly the same potential. Thus, the detector measures the voltage across the experiment plus the voltage due to noise current passing through the finite resistance of the ground bus. This is shown by  $V_G$  in Figure 3. Resistive coupling can be cured by grounding everything to the same physical point.

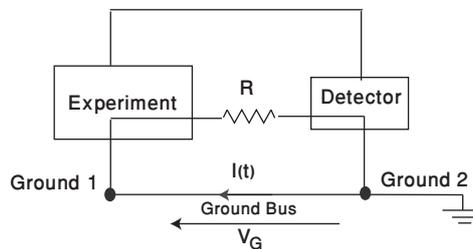


Figure 3: Resistive Noise Coupling.

Other noise sources include mechanical vibrations.

Laboratory practices can reduce noise sources to a manageable level and the lock-in amplifier employing the technique of phase sensitive detection can be used to recover signals which may still be buried in noise.

### 3.2 Internal working of the Lock-in Amplifier

The lock-in amplifier is used to detect and measure very small AC signals. A lock-in amplifier can make accurate measurements of small signals even when the signals are obscured by noise sources which may be thousand of times larger. Essentially, a lock-in is a filter with an extremely narrow bandwidth which is tuned to the frequency of the signal. Such a filter will reject most unwanted noise to allow the signal of specific frequency to be measured. For complete details the reader is referred to [3]. Figure 4 shows the approximate internal schematic of a typical lock-in amplifier.

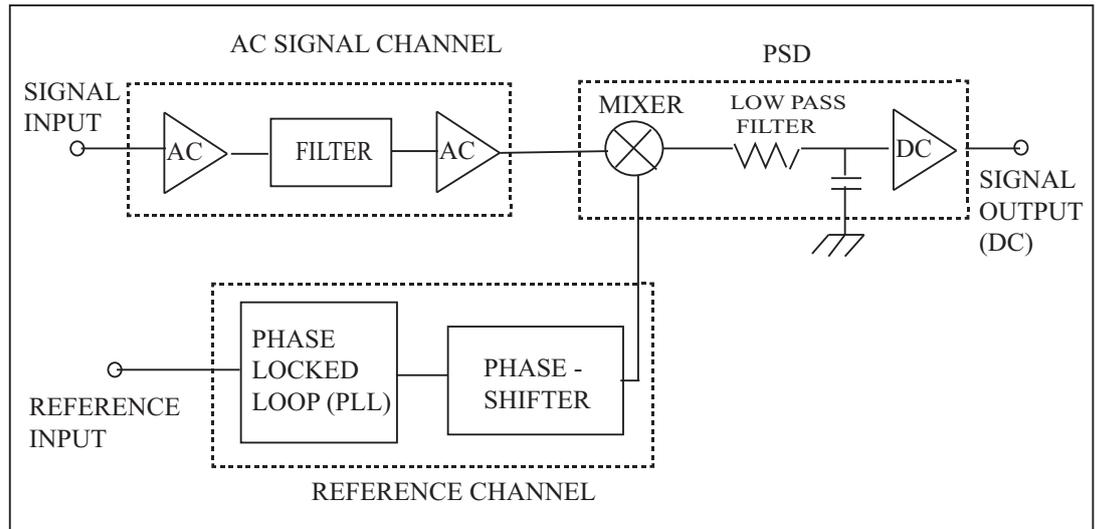


Figure 4: Inner schematic of a Lock-in Amplifier.

The lock-in amplifier is basically a phase sensitive detector : a mixer followed by a low pass filter. These components are explained here.

### 3.3 Mixer

The lock-in technique requires a reference frequency. We synchronize or derive the signal of interest from a suitable reference signal. PSD operates by multiplying two signals together.

The demodulation process is analyzed in two ways: graphically as well as mathematically. In the graphical method, illustrated in Figure (5), different possibilities for the signal are shown. The left most signal (a) is in phase with the reference signal and the right most (c) is  $180^\circ$  out of phase with the reference, while the center one (b) is  $90^\circ$  out of phase. First, consider the case (a). During first half cycle the reference is positive, the mixer output is positive. In the negative half cycle of the reference, the mixer output is again positive. Therefore, when the reference and sinusoids are in phase, the mixer output is a full wave rectified sinusoid, whose dc component is proportional to the input signal.

★ Q 1. Explain the output when the input signal is  $90^\circ$  and  $180^\circ$  out of phase with the reference signal.

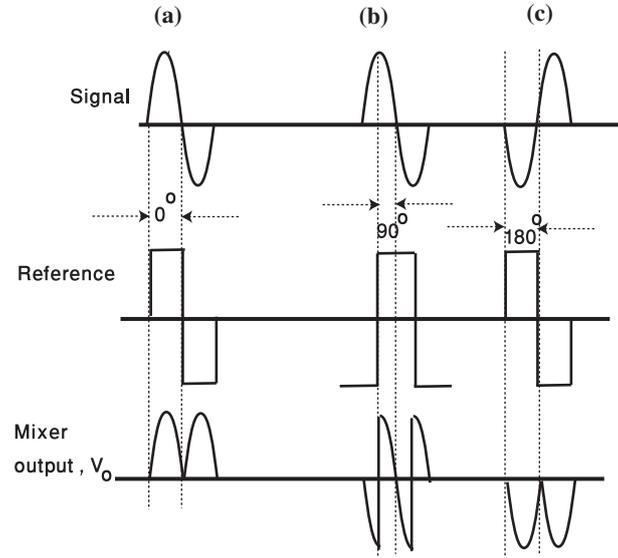


Figure 5: The output  $V_o$  is determined by multiplying the signal and the reference wave.

We can also describe the mixing operation mathematically. Consider a sinusoidal input signal,

$$V_{in} = A \sin \omega t \quad \text{with} \quad \omega = 2\pi f, \quad (1)$$

where  $f$  and  $\omega$  are the angular frequencies of the signal. A square wave can be represented as the sum of odd harmonics of the sinusoid. Suppose we have a reference signal of amplitude  $B$ , frequency  $\Omega$  at phase  $\phi$  relative to the input signal and considering the fundamental only,

$$V_{ref} = B \sin(\Omega t + \phi). \quad (2)$$

The mixer operates by multiplying the two signals together so, the output  $V_o$  will be,

$$V_o = A \sin(\omega t) B \sin(\Omega t + \phi) \quad (3)$$

$$= \frac{AB}{2} (\cos((\omega - \Omega)t + \phi) - \cos((\omega + \Omega)t + \phi)), \quad (4)$$

showing that the mixer output comprises two AC signals, one at the difference frequency  $(\omega - \Omega)$  and other at the sum frequency  $(\omega + \Omega)$ . If reference frequency is equal to the frequency of input signal i.e.  $\omega = \Omega$ , a sinusoidal output is obtained with some DC offset Figure (5).

$$V_o = \frac{AB}{2} (\cos(\phi) - \cos(2\Omega t + \phi)). \quad (5)$$

So, the output  $V_o$  is proportional to the magnitude of input signal  $A$ , the cosine of angle between input and reference and it is modulated at twice the reference frequency.

### 3.4 Low Pass Filter

If the output voltage from the mixer,  $V_o$  is passed through a low pass filter whose cut off is  $\ll 2\Omega$ , the sinusoidal component is removed and we are left with the DC part only,

$$V_o = \frac{AB}{2} \cos(\phi). \quad (6)$$

Noise close to reference frequency contributes to a small magnitude to mixer output. The noise rejection depends upon the low pass filter bandwidth and the roll-off. As the bandwidth is made narrower, the noise rejection is improved.

## 4 Finding a weak optical signal buried in noise

### 4.1 Objective

In this section, students will detect a weak optical signal using an optical chopper and a lock-in amplifier.

When two polarizers are placed nearly crossed with each other, we get a very small signal on the photodetector which cannot be measured by an ordinary voltmeter. The lock-in amplifier can be used for the detection of such small signals, which are close to but not quite zero. This extremely small signal will otherwise be washed out by ambient light.

The signal coming from the continuous wave laser is a DC signal whereas we require a modulated signal for the lock-in to operate. For this purpose, we use an optical chopper.

### 4.2 Optical Chopper

Modulation is the key step that enables the use of the lock-in amplifier. Modulation is the conversion of the DC signal to an AC signal with a defined frequency. In the present experiment, an optical chopper is used to square wave modulate the intensity of an optical signal. It comprises a chopping wheel, a motor and a speed control mechanism. The chopping wheel/blade is a rotating metallic disk with slots cut in the blade. The speed and number of slots determine the modulating frequency of the chopper. The modulation not only differentiates against noise but also discriminates against background light of constant intensity. Our optical chopper, SR540 SRS, also provides an output frequency equal to the modulation frequency, which serves as the reference frequency for the lock-in amplifier.

### 4.3 Procedure

Carry out the following procedure to measure a weak signal (obtained by crossing two polarizers), using the lock-in amplifier and optical chopper.

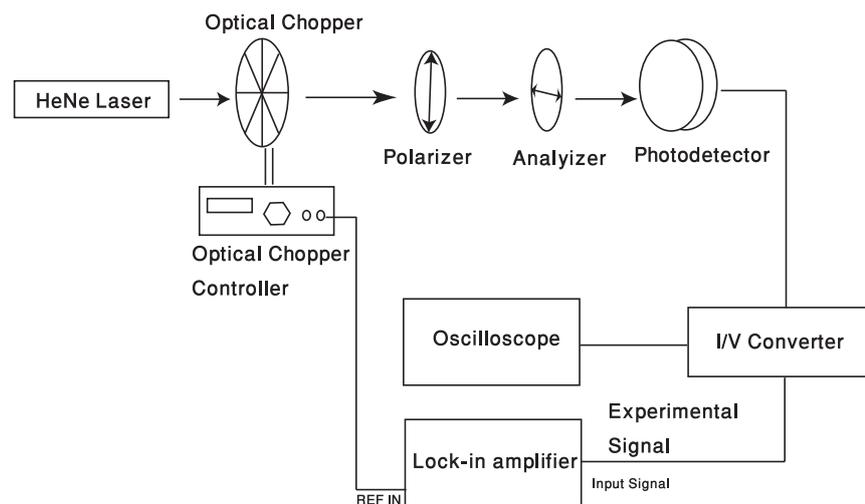


Figure 6: Weak Signal Measurement.

1. Connect the setup as shown in the Figure 6.
2. Switch on the optical chopper (study [4] before operating). Set the frequency to around 230 Hz. Keep the polarizers nearly crossed. The reference signal for the lock-in amplifier is provided by the chopper.

★ **Precaution** Always check the “OVL D” blinking on the lock-in amplifier. If it does, adjust the ‘sensitivity’ button.

★ **Q 2.** Connect the photodiode’s output to the provided I/V convertor. Observe the I/V’s output and the reference signal on the oscilloscope operating in the ‘dual mode’. Describe your observations.

★ **Q 3.** Adjust the polarizers to bring about a peak output voltage of 100 mV from the photodiode, as observed on the oscilloscope. Then, connect the signal to the lock-in amplifier. Adjust the sensitivity, pre and post filters time constants and find the output. The lock-in amplifier shows an rms value in volts.

★ **Q 4.** Find the Fourier series of the I/V’s output of Q2 and find the coefficient of the fundamental frequency. Correlate it with the measurement on the lock-in amplifier of Q3.

★ **Q 5.** Calculate the current being produced by the photodiode using the result of Q3.

★ **Q 6.** Now, connect the photodiode’s output to the current input of the lock-in amplifier. Use the  $10^6$  V/A conversion factor to find the current. Verify the two readings.

★ **Q 7.** Now, cross the polarizers rendering the output on the oscilloscope immeasurable. Then, connect the output to the lock-in amplifier and see if you can still measure the output voltage. This step should be the most revealing aspect of the lock-in amplifier: its ability to detect a feeble voltage which is otherwise immeasurably small. Fully describe your observations and the settings of the lock-in amplifier.

★ **Q 8.** Find the output of the lock-in amplifier when the **REFERENCE MODE** is adjusted to  $2f$ . What does the output represent?

★ **Q 9.** In Q7, what is the optical power of the laser light after passing through the crossed polarizer? You may need to consult the responsivity curve of the photodetector [5].

★ **Q 10.** Find the output voltage using lock-in amplifier at the chopping frequency of 100 Hz. Can you explain the cause of the strange reading at this frequency?

## 5 Finding an unknown resistance

There are essentially two ways to measure the electrical resistance of any device: sending in a known current and measuring the resulting voltage across it or applying a known voltage and measuring the resulting current through it. Technically the former option is easier.

★ **Q 11.** Find the theoretical value of  $R$  for a lead-tin alloy of length  $L$  and diameter  $d$ . The lead-tin alloy is the commonly used alloy in jumper wires. Resistivity of the tin alloy is approximately  $0.109 \mu\Omega m$ .

★ **Q 12.** Set up the apparatus according to Figure 7. Generate a 1 Volt, 80 Hz sine wave using the signal generator. Now using the lock-in amplifier, find  $V_{wire}$ .

★ **Q 13.** Calculate  $R_{wire}$  using the following expression,

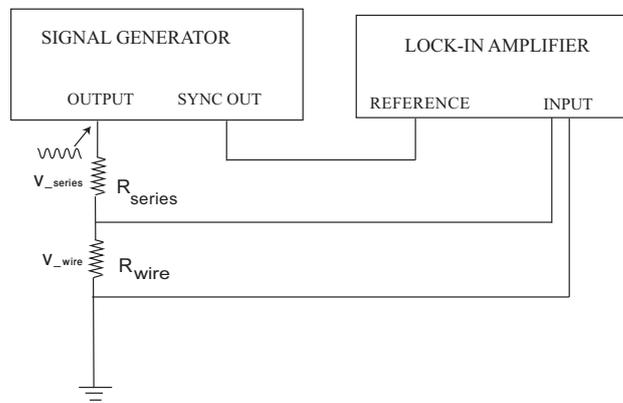


Figure 7: Circuit Diagram.

$$R_{wire} = \frac{R_{series} V_{wire}}{V_{in} - V_{wire}}. \quad (7)$$

- ★ **Q 14.** Derive the above expression using the approximation  $R_{series} \gg R_{wire}$ .
- ★ **Q 15.** Compare the theoretical and experimental values of  $R_{wire}$ .
- ★ **Q 16.** Measure  $R_{wire}$  at very low frequencies. Explain the results. Hint: The measurements are much noisier at low frequencies because of  $1/f$  noise.