Magnetic Susceptibility and the Superconducting Transition

By the end of this lab you should be able to:

- Explain what inductance is and how it relates to Faraday's Law of induction.
- Calculate the inductance of a solenoid coil.
- Explain what permeability and magnetic susceptibility are.
- Measure AC impedance, and use to determine inductance of a coil.
- Discuss the difference between ferromagnetism, diamagnetism, and paramagnetism.
- Relate diamagnetism to superconductivity.
- Explain how to measure the transition temperature of a superconductor using its diamagnetism.

Expected timeline: The first activity will acquaint you with inductance and AC measurement. This activity is expected to take 1 week. The second activity is to measure the transition temperature of a high-Tc superconducting sample, and will require a second week.

Pre-lab questions on D2L must be submitted prior to Activity I.

In-class quizzes will be given each period, and will help you with explanations, diagrams, etc. that go in your final report.

Final report: will be due one week after the end of the lab. An online template and grading rubric will be provided.

Activity I:
Equipment needed:

- ✓ Function generator
- ✓ 2 DMMs
- ✓ Film canister/magnet wire
- ✓ 100 Ω resistor, wires, tools
- ✓ Hot glue gun

A. Problem:

You have developed an unknown magnetic sample, and wish to determine its relative permeability. You wish to measure the permeability and susceptibility by wrapping the sample in a coil with known air-core inductance, and then measuring the ac reactance of the coil with sample inserted.

B. Terms to define

Almost any introductory physics text will have descriptions of the following terms.

Inductance (symbol L)
AC Reactance (symbol X)
Impedance (symbol Z)
Permeability (symbol μ) and relative permeability (μ/μ₀, subscript denoting air-value))
Susceptibility (symbol χ)
Paramagnetism, ferromagnetism, and diamagnetism

C. Relationships to become familiar with

Know how these relations were developed from first principles.

1. Faraday's Law of induction:

$$v = L\frac{di}{dt}$$

2. Inductance of a solenoid (long coil):

$$L = \mu N^2 \frac{A}{l}$$

N is the number of turns in the coil;
A is the area of the coil;
l is the length of the coil.

3. Impedance and relationship to reactance (complex quantity):

$$Z = R + jX$$

Here R is resistance, a real number always;

X is a combination of capacitive and inductive reactances, and is imaginary. The imaginary nature of the reactance comes from representing an AC signal as a complex exponential function. So, for example, assume AC current *i* is a cosine function:

$$i = i_0\left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right).$$

Then Faraday's Law of induction (eq. 1.) becomes

$$v = i_0 j\omega L\left(\frac{e^{jwt} - e^{-jwt}}{2}\right).$$

Thus, in an AC setting, the time-independent portion of the ratio of *v* to *i*, is equal to jωL if there is just an inductor impeding the flow of AC current.

4. Impedance with an inductor

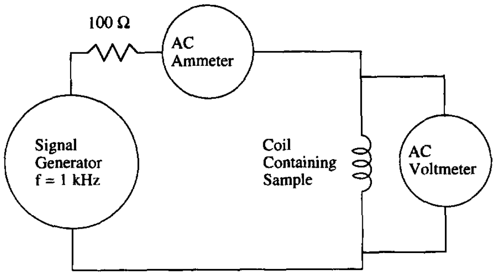
An inductor, being a coil of wire, will have both resistance and reactance to a flow of AC current. If you measure the amplitude of AC current through an inductor simultaneously with the amplitude of AC voltage, the ratio *v/i* gives the magnitude of Z. Since Z is complex, the magnitude of Z is:

$$|Z| = \sqrt{R^2 + X^2} = \sqrt{R^2 + (\omega L)^2}$$

This relationship, with Z measured at a known frequency ω and R measured with an ohmmeter, will allow one to determine an unknown L. Knowledge of L would allow one then to extract the permeability constant using the parameters N, A, l of the inductor coil.

D. Suggested experiment

- To make a coil of known dimensions, tightly wrap magnet wire around an empty film canister. Use a hot glue gun to anchor the coils onto the canister so they don't unravel when no longer held. The canister will later hold an unknown sample for which you will determine permeability and susceptibility.
- Carefully determine the resistance R of the coil.
- Determine the inductance L of the coil in three ways: by calculating L from the coil dimensions and assuming an air core; by using the inductance bridge available in the lab; by measuring Z of the coil when AC current flows through the coil. A frequency generator may be used to create the AC current, but place a 100 Ω resistor in series with the generator output so that the coil does not short the output of the generator. Two multimeters may be set up to measure the amplitude of the current through and voltage across the coil. Z can be determined using a graph of voltage amplitude versus current amplitude.
- Refer to attached article from The American Journal of Physics by H. G. Lukefahr et al, *A very simple and inexpensive apparatus for detecting superconducting transitions via magnetic screening* (**Am. J. Phys. 65** (2), Feb. 1997, p. 132). Fig. 1 in that article shows a circuit schematic, shown below for convenience. Other parts of that article may also be helpful at this point, so be sure to examine the article. You will revisit the article in Activity II.



- Be sure to try more than one frequency of AC in order to determine L. A frequency close to 5000 Hz is suggested, and try at least one more near 3000 Hz.
- Be sure to discuss in your report of findings whether all three measurements

of L with the air core agreed with each other, and try to account for differences.

- Obtain an unknown sample inside the film canister. Again, measure the new inductance using both an AC bridge as well as an AC measurement at one of the frequencies used previously. From these measurements, determine the inductance with the magnetic core, and extract the permeability and magnetic susceptibility for your unknown sample. Your report will need to discuss agreement between values found using the two methods.
- Was your sample diamagnetic, paramagnetic, or ferromagnetic? Be sure to discuss in your conclusion.

Activity II:

Equipment needed:

- ✓ High –Tc inductance probe
- ✓ Thermocouple and reader
- ✓ Function generator
- ✓ 2 DMMs
- ✓ 100 Ω resistor, wires, tools
- ✓ Sand cryostat components
- ✓ Booklet: Experiment Guide for Superconductor Demonstrations

A. Problem:

Your group has developed a High-Tc superconductor. Your goal is to measure the critical temperature of the sample. You are aware that a superconductor becomes perfectly diamagnetic (magnetic susceptibility = -1) below the critical temperature, hence the ability to levitate magnets above the sample when it has been cooled below this transition temperature. Thus, you believe the superconducting transition will become apparent by measuring the inductance of a coil wrapped around the sample as the temperature of the sample varies.

B. Become familiar with the experiment.

The American Journal of Physics article by H. G. Lukefahr et al, *A very simple and inexpensive apparatus for detecting superconducting transitions via magnetic screening* is attached (**Am. J. Phys. 65** (2), Feb. 1997, p. 132). Please become familiar with the experiment and background. It is also helpful to think through the question: what happens to the inductance of a coil if it surrounds a perfectly diamagnetic material? Does the inductance decrease, stay the same, or increase?

C. Some background on working at low temperatures

High-Tc superconductors were developed to have transition temperatures near that of boiling liquid nitrogen, -196 deg. C or 77 K. Thermocouples are the most reliable way to measure such low temperatures. Since you will be using a thermocouple to measure the temperature of your sample, become familiar with what they are and the rudiments of how they work. You will be provided with a Type-T thermocouple, having a copper-constantan junction, embedded in the sample. You will also have a hand-held thermocouple reader to read the thermocouple temperature. Be sure the reader is as far away from the liquid nitrogen/cryostat as is possible as circuits in the reader can be affected by temperature.

The sample you will examine is BCCO, a type of ceramic known as a perovskite. All the samples in the kit are fragile, so please be careful when handling them. Even though the sample you will examine is wrapped in a coil, fractures can happen if it is dropped or cooled too quickly.

To cool the sample slowly and keep it relatively stable near the transition temperature, you will place the sample inside a sand cryostat (Fig. 1). The probe resides in the inner metal canister of the cryostat, covered completely with sand. Liquid nitrogen is poured slowly into the chamber, and can safely be poured into the sand as well as the area between the canister and outer chamber wall. Typically, you want to pour in the nitrogen just a little at a time, and monitor the temperature of the sample. When the sample is a few degrees below the transition temperature, stop adding liquid nitrogen and begin monitoring the inductance of the sample.

D. Suggested experiment

- You are familiar already with measuring inductance using an AC circuit from Activity I. This activity has you repeat the experiment but with an unknown high-Tc sample and at an AC frequency of about 5000 Hz. You will also have to make multiple measurements of L as the voltage/current change with temperature. Thus, instead of plotting *v* versus *i* to make a single measurement of L, each temperature will have its own unique *v, i* and L.
- Another different feature about this experiment is that you cannot simply use an ohmmeter to measure a single R, as R will also be changing with temperature. Instead, after you have measured the AC *v* and *i* as the sample warms up from the superconducting state, you will need to cool the sample down one more time. As the sample warms up through the transition temperature, record the resistance using an ohm-meter.
- The DC measurements can be used to determine via curve fitting how R depends on temperature. Thus, you can determine R for each of the AC values, and thus L can be calculated.
- Your final graph will show L as a function of temperature. Use the graph to determine the transition temperature. Also, discuss the diamagnetism of your sample.

E. Further Reading

The search for high-Tc superconductivity and understanding these materials makes for fascinating reading. Some background is available in the experimental booklet accompanying your probe. However, a more detailed account may be found at:
http://nobelprize.org/nobel_prizes/physics/laureates/1987/bednorz-lecture.html
(In 1987, two German materials scientists, Bednorz and Muller, received the Nobel Prize for their discovery that perovskites were high-Tc materials. Their Nobel lecture describes their discovery process.)

You may wonder why your sample does not exhibit a sharp, single transition temperature but has a region over which the transition occurs. Martin Nikolo, in section IV of his article *Superconductivity: A guide to alternating current susceptibility measurements and alternating current susceptometer design,* gives good physical insight into why this happens (Am. J. Phys. 63, January 1995, pp. 57-65.)

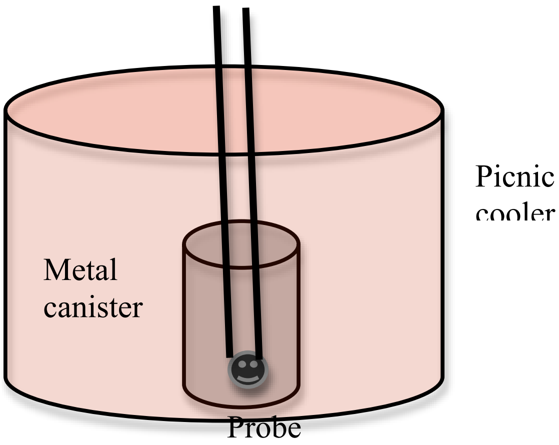


Fig. 1. Sand cryostat consisting of outer chamber (picnic cooler) and inner chamber (metal canister). The fragile probe is placed in the metal canister and completely covered with sand. Then liquid nitrogen is slowly poured in.

Instructor Notes

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From one instructor's experience, this is what students "get" from doing this lab.

- A better understanding of inductance
- Practical experience with working at low temperature
- Practical experience with working with small electrical (mV) signals
- First exposure to differences in magnetic materials—diamagnetism versus ferromagnetism
- Grappling with magnetic characterization quantities: permeability and susceptibility.

Equipment notes:

1. The 100-Ohm resistor placed in series with the coil should be used to obtain the current in the circuit, rather than an ammeter. We find that measuring the voltage across the resistor, rather than the current in the circuit, has some advantages. First, you avoid students connecting an ammeter into the circuit incorrectly (and thus blowing fuses). Secondly, the current is the order of a few mA, and most "student" ammeters do not measure such small currents with sufficient resolution. By using the resistor, the voltage across the resistor is sufficiently large to give 3 sig. figs. to the current.
2. The instructions that come with the superconducting sample suggest using an AC frequency of 1000 Hz. The sample data shown here was taken with a frequency near 1000 Hz, and you can see why we have changed the frequency to a slightly higher value. At 1000 Hz, the reactance of the coil is small compared to the dc resistance of the coil. By increasing the frequency, the reactance approaches the value of the resistance.
3. The coils made from a film canister typically have a measured inductance near 100 μH. The calculated value is typically larger than the measured value, often by more than 10%. Students usually do not come up with a reason on their own for why there is a discrepancy, especially if they have not deduced where the formula for the inductance comes from. The formula given assumes an infinitely long solenoid, whereas what they have constructed from the canister is far from "infinite".
4. When the film canister coil is filled with steel pieces—typically very small steel washers—students easily determine that the material inside is ferromagnetic. A typical measured value for the relative permeability of the steel washers is close to 2.

Difficulties or obstacles our students face with this experiment at the sophomore level

- Difficulty or unwillingness to read the AJP article suggested.
- Frustration with waiting for the sample to cool down/warm up.
- Still struggling with wiring up a simple circuit and getting it to work.
- Failure to compare the transition temperature to literature value.
- Focus on just taking data without much curiosity. (There are many things to be curious about—how does a thermocouple work? Why does a magnet levitate above a superconductor? Why is the transition temperature spread out over a range of temperatures? What does BCCO stand for? How is the sample made?) This last obstacle may be due to the fact that this experiment is quite rich in physics, and the students have too short a time to adequately address all the issues.

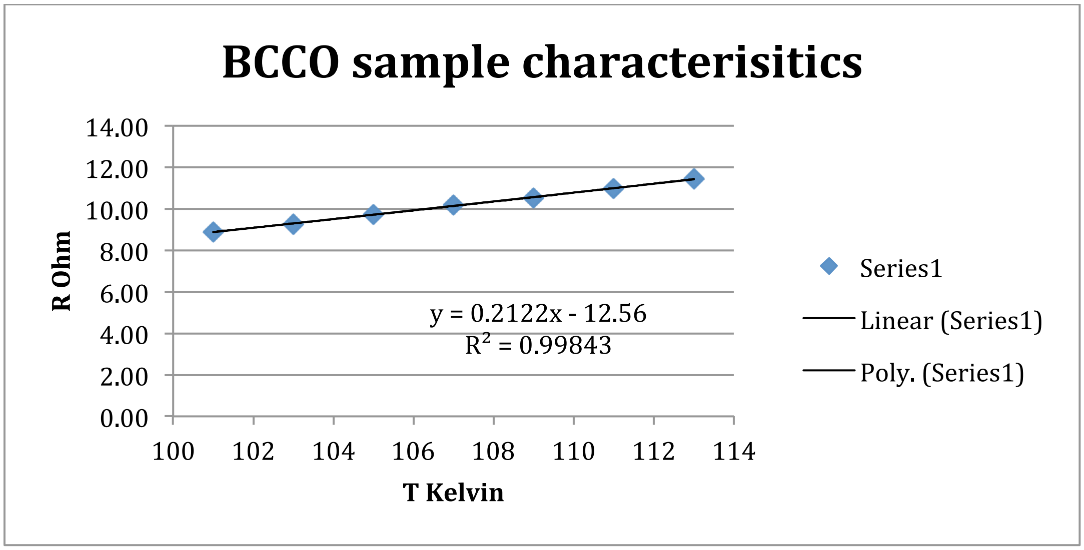
SAMPLE DATA

I. DC data on the resistance of the coil as a function of temperature.

T (deg C)	V (mV)	I (mA)	T (Kelvin)	dc R (Ohm)
-190	48	8.48	83	5.66
-188	50	8.51	85	5.88
-186	52	8.53	87	6.10
-184	53	8.54	89	6.21
-182	56	8.54	91	6.56
-180	58	8.55	93	6.78
-172	76	8.54	101	8.90
-170	79	8.53	103	9.26
-168	83	8.54	105	9.72
-166	87	8.54	107	10.19
-164	90	8.56	109	10.51
-162	94	8.56	111	10.98
-160	98	8.56	113	11.45

Last 8 data points allow a temperature-dependent model of the DC resistance of the coil to be developed.

$R(T)=0.212T-12.6$ in Ohms if T is in Kelvin



II. AC data on the impedance of the coil as a function of temperature.

Function generator f=931+/-1 Hz

T (deg C)	V (mV)	I (mA)	Z (Ohm)	Rdc (Ohm)	T Kelvin	wL(Ohm)
-174.5	73	8.31	8.8	8.34	98.5	2.75
-173.1	75	8.3	9.0	8.64	99.9	2.65
-171.7	78	8.29	9.4	8.94	101.3	2.95
-170.4	81	8.28	9.8	9.21	102.6	3.29
-169.9	82	8.27	9.9	9.32	103.1	3.39
-168.8	86	8.26	10.4	9.55	104.2	4.14
-168.2	88	8.25	10.7	9.68	104.8	4.48
-167.8	89	8.25	10.8	9.76	105.2	4.59
-167.3	91	8.24	11.0	9.87	105.7	4.96
-166.9	92	8.23	11.2	9.95	106.1	5.09
-166.5	93	8.22	11.3	10.04	106.5	5.22
-166	96	8.21	11.7	10.15	107.0	5.81
-165.5	97	8.21	11.8	10.25	107.5	5.87
-165	99	8.2	12.1	10.36	108.0	6.20
-164.5	102	8.2	12.4	10.46	108.5	6.73
-164	103	8.19	12.6	10.57	109.0	6.81
-163.5	106	8.18	13.0	10.68	109.5	7.34
-163	107	8.18	13.1	10.78	110.0	7.41
-162.5	109	8.18	13.3	10.89	110.5	7.68
-162	110	8.18	13.4	10.99	111.0	7.74
-161.5	111	8.18	13.6	11.10	111.5	7.81
-160.4	113	8.17	13.8	11.33	112.6	7.93