The Swinging Atwood's Machine: An Experimental Approach

Physics 335: Advanced Laboratory

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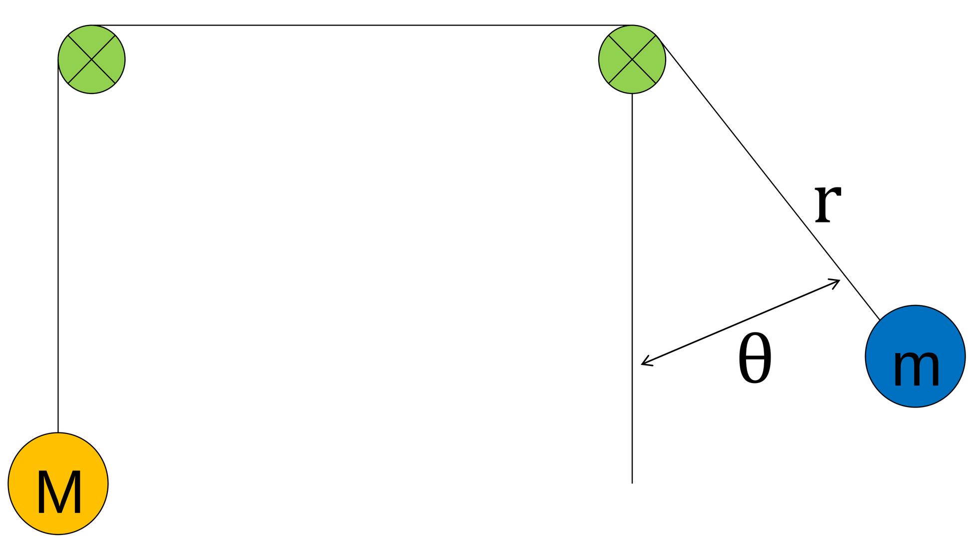
Abstract

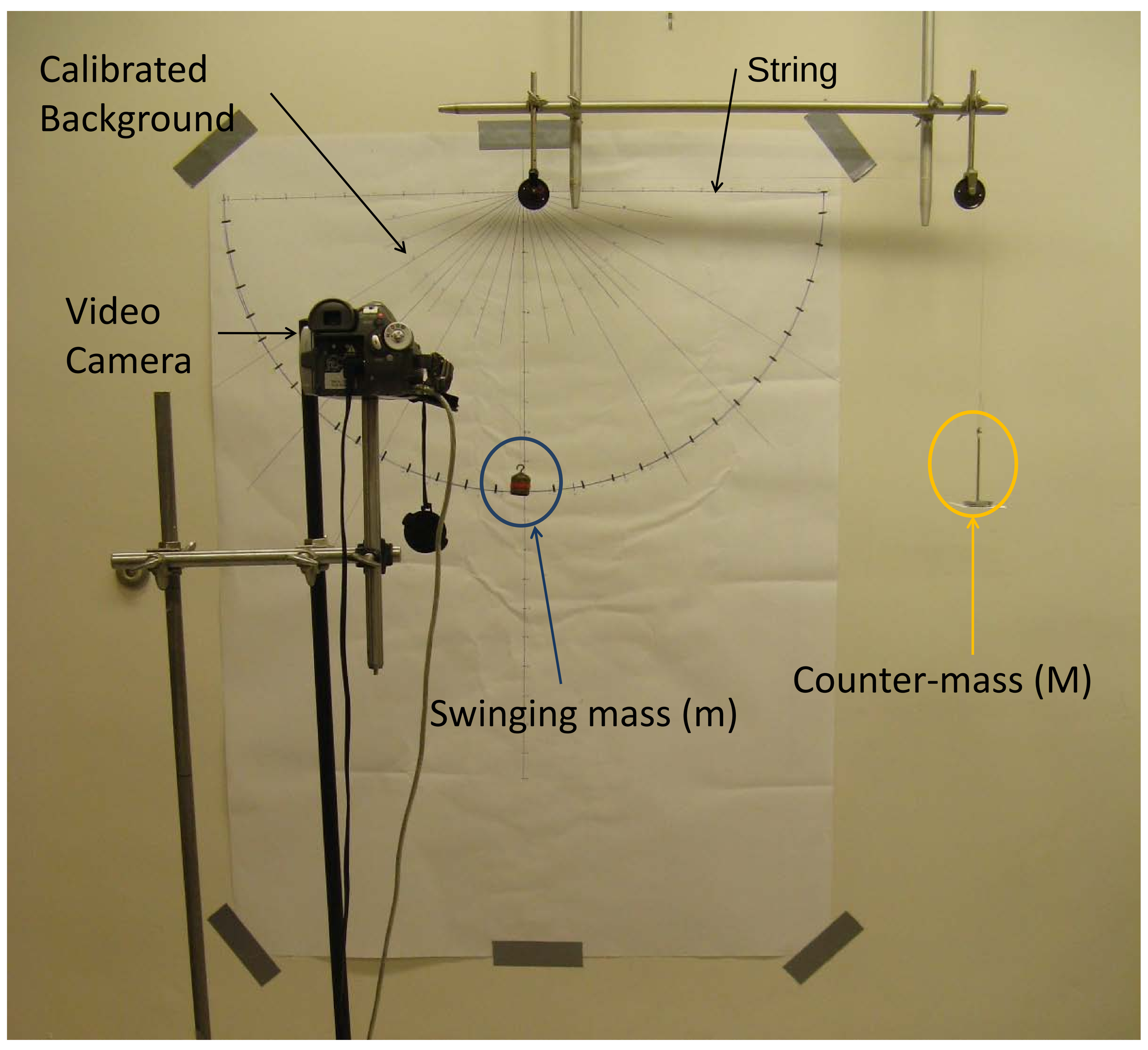
The swinging Atwood's machine is an extension of the introductory physics problem in which two masses are connected by a string over an ideal pulley. In this extension of that problem, one mass is displaced by an angle theta. By displacing the mass by theta, the tension of the connecting string changes (unlike the standard problem) as the one mass swings which causes the other mass to move up and down. By changing the ratio of the two masses and the initial angle of displacement, the trajectory of the swinging mass changes. In this experimental approach, we examine the necessary angle to mass-ratio relationship necessary to produce pendulum-like motion in the swinging mass. This is achieved through the use of video recording and object tracking software. Our experimental results will be compared to the Java-based computational simulation developed by the same author.

Introduction



The swinging Atwood's machine (SAM) is an extension of the simple Atwood machine. Although the SAM is typically studied theoretically or computationally, some work has been done experimentally. The physical system is, however, limited by the nature of the pulleys involved and altered by non-ideal forces such as friction.

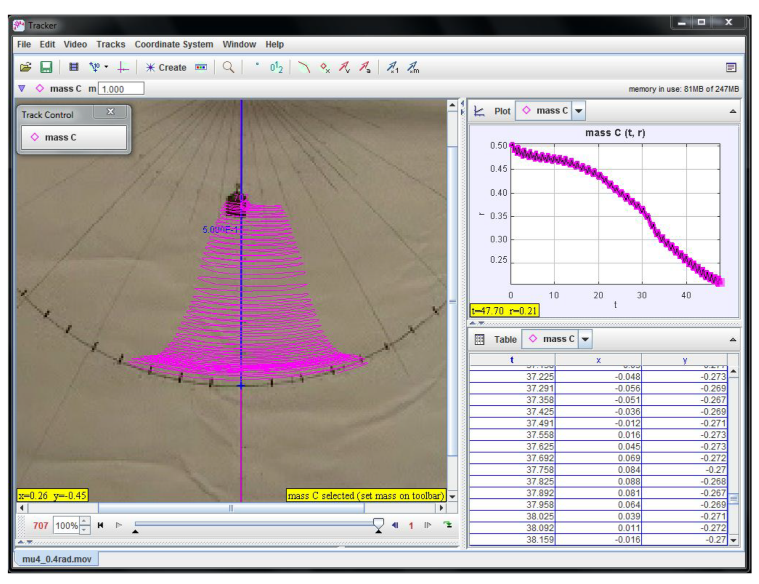
Apparatus and Procedures



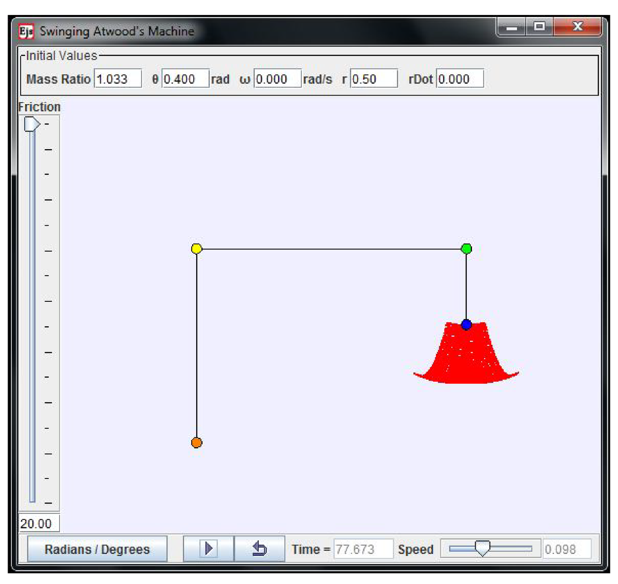
Masses Used. The swinging mass, left, is brass and filled with sand. The counter-mass, right, can be given more or less mass by adding or removing calibrated masses.

The Apparatus. Two pulleys are clamped to the wall with a thin, non-elastic string between them and a mass on both ends. Behind the swinging mass is a calibrated background, which is calibrated vertically in 10-cm increments, and in the angular direction in 0.1-rad increments at a radius of 50 cm. A video camera records the motion of the swinging mass.

Taking Data. On every data set, the swinging mass was displaced a certain angle θ with a radius of 50 cm. We paid special attention that the mass was released as straight as possible. The motion was then recorded using a video camera and Debut Video Capture Software. That data was then analyzed using Tracker Video Analysis Software. Final data analysis was done using Origin 7.5 Data Analysis and Graphing Software. The experimental results were compared to computation results generated in Easy Java Simulations (EJS).



Tracker Screen Shot



EJS Screen Shot

Theory

The Lagrangian, which is equal to the kinetic energy minus the potential energy (L = T-V), is:

$$\mathcal{L} = \frac{1}{2} M \dot{r}^2 + \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2\right) + gr\,(m \cos\theta - M)$$

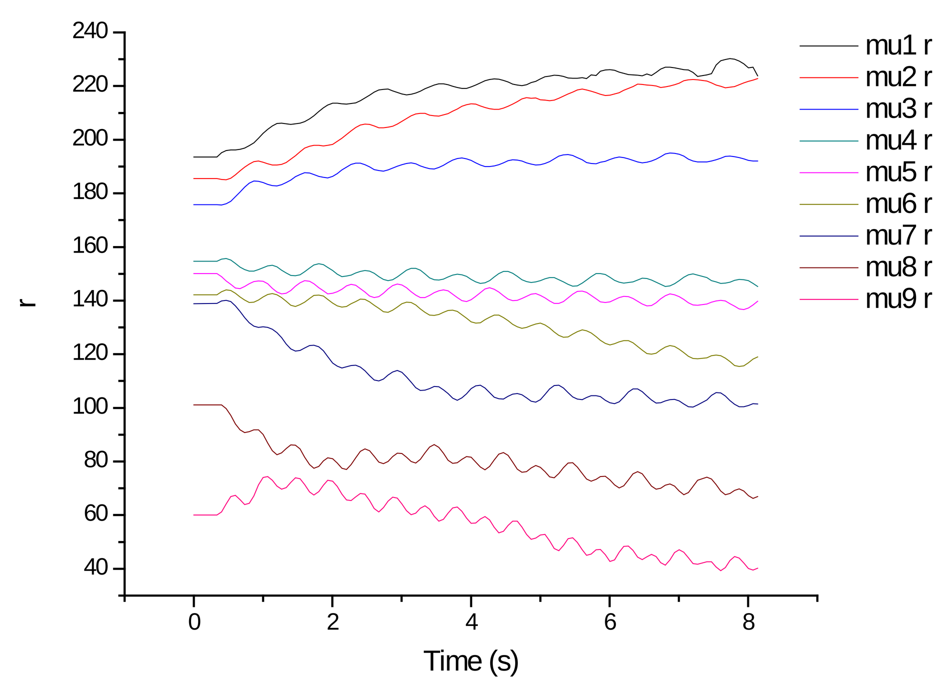
The equations of motions can then be found using the Euler-Lagrange equation. The equations of motion for the two degrees of freedom are therefore:

$$\ddot{r} = \frac{r\dot{\theta}^2 + g\,(\cos\theta - \mu)}{(1 + mu)} \qquad \ddot{\theta} = \frac{2\,\dot{r}\,\dot{\theta} + g \sin\theta}{r}$$

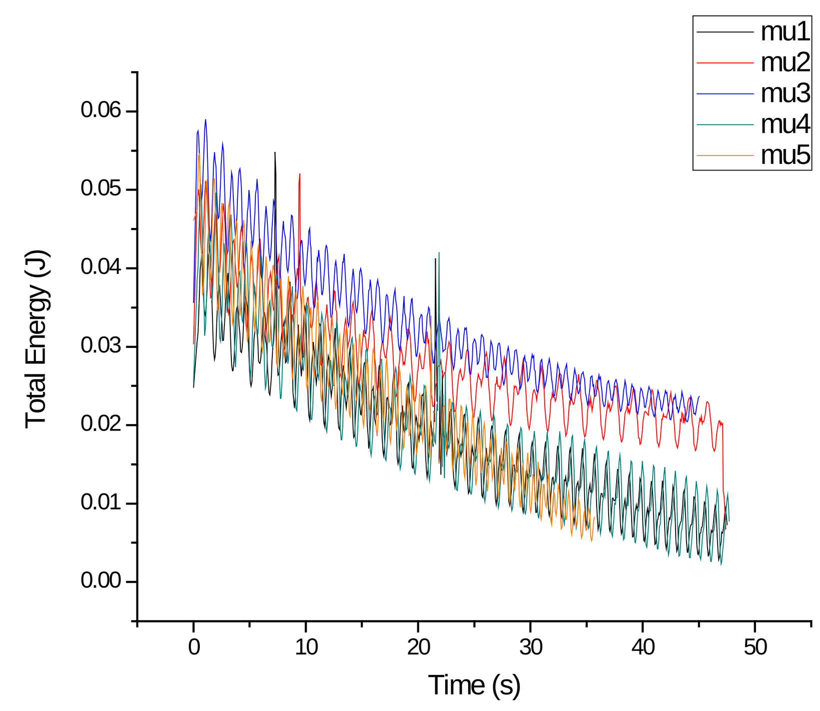
Where $\mu = M/m$.

Results

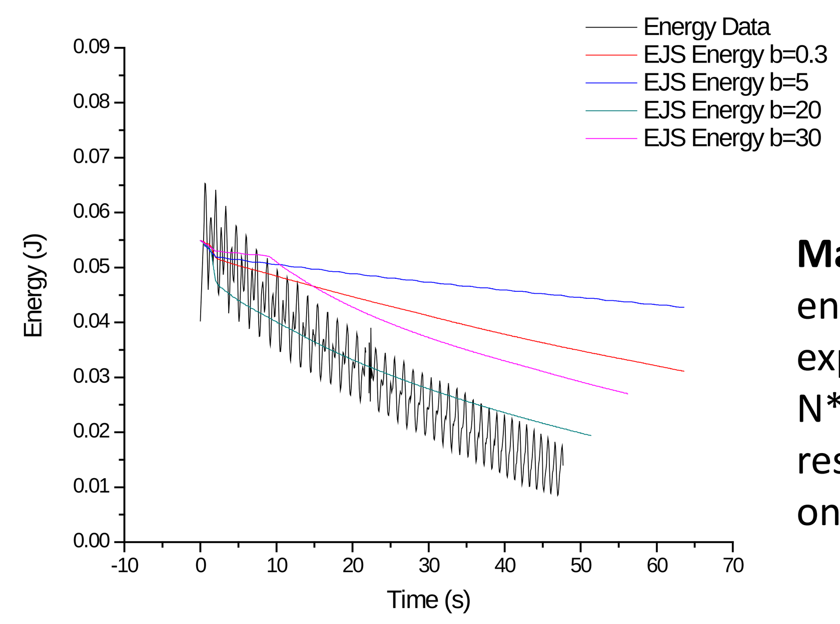
All results are for $\theta_i = 0.4$ radians



The r(t) Dependence on mu. The length r changes over time and is impacted by mu. For any given initial angle, there is an ideal mass ratio in which the r(t) will have neither an upward nor downward trend. That motion would be a pendulum-like path



The E(t) for Different mu Values. The general downward trend of the energy over time shows the effect of friction on the system. Friction, a non-conservative force, bleeds energy out of the system.



Matching the Experimental Data with EJS Data. EJS energy calculations are compared to the experimental data. A damping constant of b=20 N*s/m appears to be closest to the experimental results. This was done to gauge the effect of friction on the system.

Conclusion

In general, we found the motion of the swinging Atwood's machine to be consistent with the expected results. As the swinging mass oscillates, the counter-mass moves up and down. There is also a specific, ideal mass-ratio for an initial θ in which the swinging mass does not have a general upward or downward trend.

However, forces and factors not built into the ideal system have a larger effect on the system than anticipated. It is possible that the moments of inertia of the pulleys, although small, had a non-negligible effect on the system. Friction also bled off a significant amount of energy at fast rate. Future experimental and computational work on this subject would do well to try to further minimize the effects of these factors.

References

"Swinging Atwood's Machine". Nicholas B. Tufillaro. *American Journal of Physics*. 18 Nov 1983.

"Teardrop and heart orbits of a swinging Atwood's machine". Nicholas B. Tufillaro. *American Journal of Physics*. 26 Aug 1993.

Wolfgang Christian's EJS adaption of ``An Introduction to Computer Simulation Methods: Applications to Physical Systems," by Harvey Gould, Jan Tobochnik, and Wolfgang Christian

OSP Collection on the ComPADRE Digital Library: http://www.compadre.org/osp/

Easy Java Simulations: http://www.um.es/fem/EjsWiki/

Tracker Video Analysis and Modeling Software: http://www.cabrillo.edu/~dbrown/tracker/

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