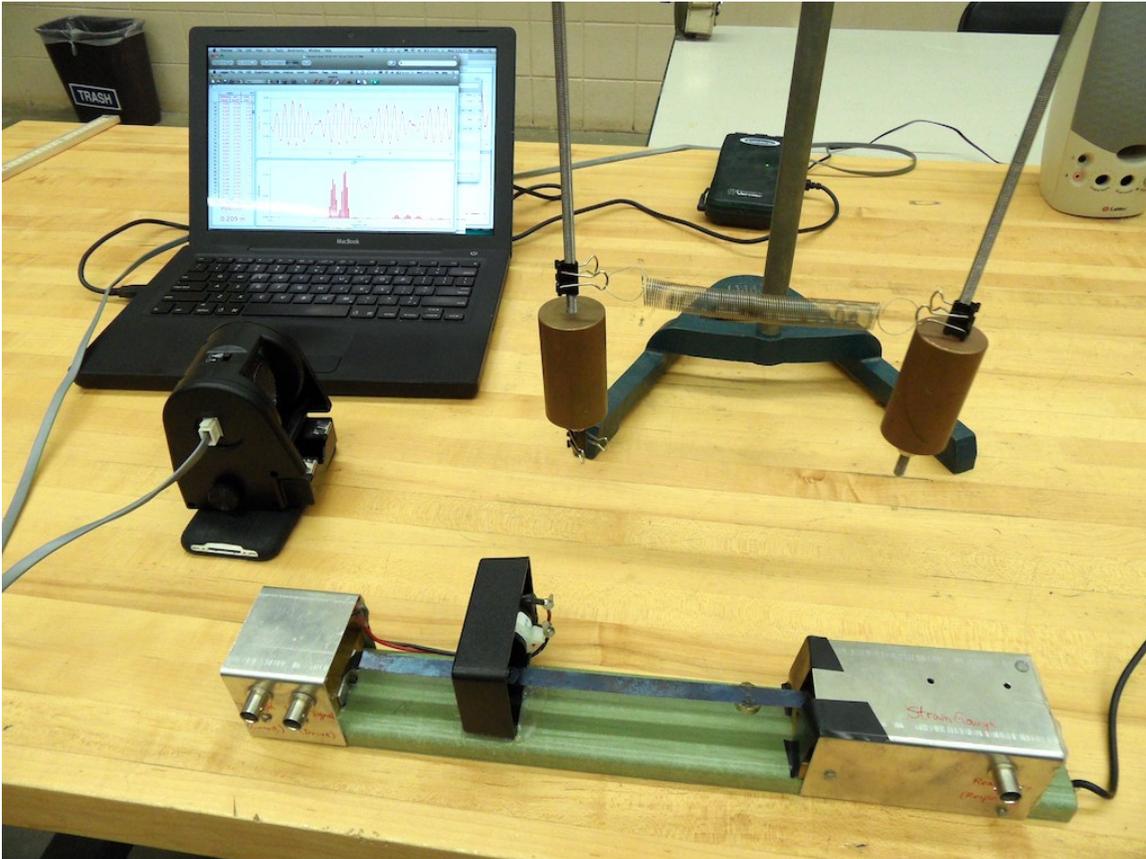


# Normal Modes

Presented by:  
Gabe Alba

Instructional Laboratory Administrator  
Department of Physics & Astronomy  
Rutgers University (Piscataway, NJ, USA)  
[alba@physics.rutgers.edu](mailto:alba@physics.rutgers.edu)



**Equipment List:** Vibrating Strip apparatus\*, small foam pads (2), small aluminum block (1), Read/Drive cable patch panel, PC sound card interface, speakers, FFTScope\* software (Mac or PC), cables; Two pendulums coupled with spring, motion sensor, supporting beam, ring stand.

\*Vibrating Strip apparatus developed and constructed by Hsu-Chang Lu, Rutgers University. FFTScope software written by Hsu-Chang Lu, Rutgers University.

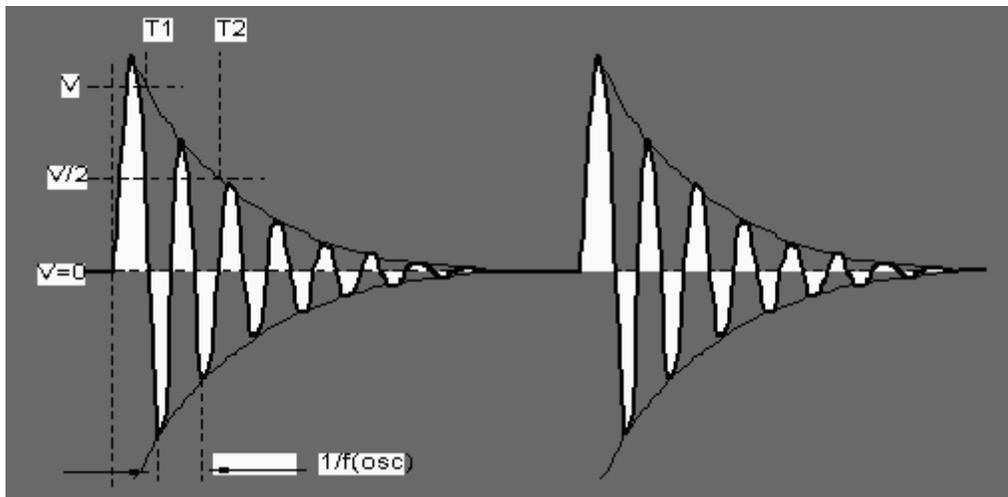
## Normal Modes

**Objective:** To investigate the behavior of a mechanical system

**Apparatus:** Vibrating Strip apparatus, small foam pads (2), small aluminum block (1), Read/Drive cable patch panel, PC sound card interface, speakers, FFTScope software, cables; Two pendulums coupled with spring, motion sensor, supporting beam, ring stand.

### Introduction

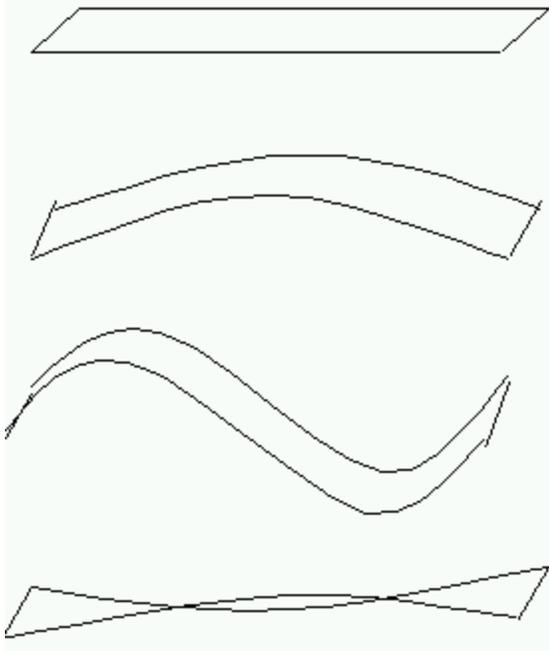
One of the last labs you performed last semester in 229 was the [RLC Circuits](#) experiment. In that lab you examined how energy was shuttled back and forth between the Inductor (L) and Capacitor (C) of the circuit, resulting in an oscillation:



Some important results you got from that lab were:

1. There is *no oscillation* if only C exists in a circuit, without L; you get only exponential decay. Both L and C are needed to exchange energy constantly.
2. The amplitude of oscillation dies down due to energy being dissipated through the Resistor (R). R provides *damping* for the oscillator.
3. The circuit provided the *maximum response amplitude* when driven at the characteristic frequency  $\omega_0$ . This is the "natural" frequency at which an oscillator likes to oscillate.
4. The *phase* or difference in synchronization between the driving voltage and the response voltage is *zero* at the characteristic frequency  $\omega_0$ . This means that if you drive a system at its natural frequency, you will get the maximum response - think of you and a friend trying to dislodge a car stuck in the snow: you move the car by pushing it at precisely the same time it rocks away from you. (If you chose to push *while* it was coming toward you, you would just kill the rocking. Think of pushing a swing, as well.

In this lab, your systems will be a mechanical, rather than electrical. It will also have various ways to oscillate (note that the circuit only had one way - for energy or charge to oscillate between L and C). The first system consists of a thin metal strip, which is fixed at both ends. It has more than one *mode* of vibration; its **normal modes** are listed below:

<p>From top to bottom, the normal modes are:</p> <ol style="list-style-type: none"> <li>1. Stationary (not moving)</li> <li>2. <u>Symmetric</u> mode (both left and right sides of the strip move in the same direction at the same time).</li> <li>3. <u>Antisymmetric</u> mode (left and right sides of the strip move in opposite directions at the same time).</li> <li>4. <u>Torsional</u> mode (twisting along the length of the strip). There may be variations of this, and the above modes.</li> </ol>	
---	---

Although the strip vibrates too quickly for your eyes to see, you can *detect* the frequencies of vibration of these modes using the FFT method you used last semester. Note that the strip has two magnets attached to its underside; they are there so that the system can be damped (through Eddy currents, by placing an aluminum slug under one magnet) and can also be driven (through magnetic induction, with the solenoid under the other one). Despite the fact that these add mass, the strip + magnets nevertheless constitute a system with a restoring force that will oscillate when energy is added. Your goal will be to find the frequencies of these normal modes of vibration listed above.

The second system is composed of two pendulums coupled together with a spring. Like the RLC circuit, the two pendulums exchange energy with each other. The system will be undriven, so you will have to get it started. Again, your goal will be to find the normal mode frequencies, without first knowing all the possible ways this system can vibrate.

### Procedure - Vibrating Strip

0. Remove foam pads under the apparatus base or magnets under the strip, if any.
1. Open "Scope Fast", which is a FFTScope template file in the same directory as this write-up. Click on the GO/STOP button to start the Oscilloscope. Gently pluck the strip

in the middle, between the two magnets - you should see a fairly smooth sinusoidal trace (symmetric mode). If the tops of those waveforms are cut off (flat-topped), you've plucked too hard.

2. Now try plucking away from the middle of the strip - you may see some additional structure to the previously-smooth sine waves. Note that the superposition of smaller waves on top of the large symmetric wave indicates that you have excited some additional modes, in addition to the symmetric mode (like asymmetric, torsional) - what you see is the **sum** of the modes you have excited.

3. Keep plucking closer to the ends of the strip - what happens to the waveform now?

4. Gently strike the strip close the right edge with a pen or pencil - can you excite a mode that minimizes the presence of the symmetric mode?

5. Close the FFTScope window and open the template file "Scope Slow". This will enable you to fit many more waveforms on the scope display at the same time, to observe how the amplitude changes with time. Pluck the strip, then press the RETURN key on the keyboard, and the SPACEBAR key immediately after that - this will enable you to observe a 5-second time window of the waveform and freeze the waveform for observation (Scope Fast only gave you a 0.1 second time window). Measure, from the display, how many seconds it takes for the oscillation to decay to half its original amplitude (measure amplitude from top to bottom by positioning cursor on wave peak and reading corresponding amplitude and time from numbers on lower left of FFTScope window).

6. Put the foam pads at both ends of the apparatus base. Repeat the measurement above. Finally, add the aluminum block - place it right under the left magnet. Repeat the measurement. Did the oscillation last longer or shorter? Explain.

7. So far you have been examining the vibrations in Oscilloscope view (Amplitude vs. Time). You are ready to see them in the FFT view (Amplitude vs. Frequency) - close the FFTScope window and open "FFT Slow". Notice that the horizontal (X) axis has a very wide range of frequencies - the oscillator will only vibrate in a range much lower than that, so click and drag your cursor horizontally to zoom in on just a fraction of that range (say, the first few hundred Hz). Click the "Turn averaging on/off" button (between the GO/STOP and OSC/FFT buttons - you will "FFT Averaging on n=1" below the button.

6. Click the GO button, then pluck the string and wait 5 seconds to see a FFT peak grow. Auto scale (button under the Help) menu item to see entire peak (ignore the large false "peak" around zero Hz). Which peak is dominant? (record the frequency after you position your cursor over it and noting the numbers in the lower left of the window) What other peaks are present? (also record)

7. Try, as you did in Steps 2-4 above, exciting various modes of the strip by gently plucking it at various parts, twist-plucking, or perhaps simultaneously releasing one end

high and the other end low with your fingers. Record the frequencies of the peaks you can excite. Do the relative heights of the peaks depend on the way you pluck the strip?

8. You must be tired of plucking by now, so let's use FFTScope's built-in Function Generator. Close the window and open the file "Scope Fast". Press F9 to enter a frequency, say 30 Hz to drive the strip (remember, a solenoid below the right magnet will flip its Magnetic Field with this frequency you select). Go to Function Generator and select Sine as the driving waveform. Feel the vibration on the strip with your finger; if not, turn the Volume knob on the speakers to full clockwise. Press GO to start the oscilloscope again. Autoscale by pressing the button with the four arrows (to the left of the button with the two arrows). Autoscale as necessary, but don't autoscale again after you change the frequency using F9. Note the peak-to-peak height of the waveform - find the frequency (27? 28? 29? 30? 31? 32 Hz?) at which this height is a maximum. Which mode does this correspond to?

9. Go back to FFT view (open FFT Slow file) and see record the frequency of the highest peak. How does this compare to the frequency you got in Step 9?

10. Now, instead of driving at a specific frequency, drive the strip with the whole spectrum of frequencies using White Noise, in which the phases are random. Select "1/100 sampling rate White Noise" in Function Generator and turn Averaging ON. Again select the appropriate frequency range by zooming in with your cursor and start taking data. Let the gathering continue for several minutes (FFT Averaging on  $n > 10$ ). Note all the frequency peaks; enlarge certain portions of the frequency range and Autoscale as necessary.

11. Click on the Stereo/Mono button to display Phase data, as well as. Note how the Phase Difference angle (right scale; Amplitude is on the left scale) changes at the normal mode - what happens to it at these frequencies?

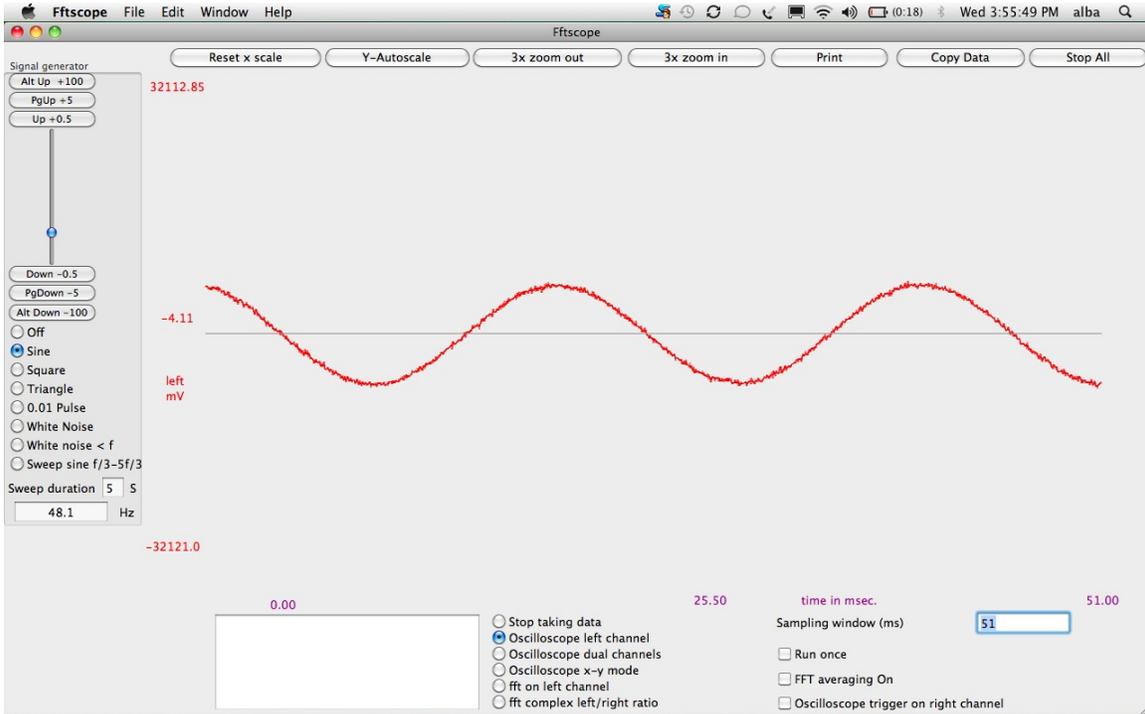
### **Procedure - Coupled Pendulums**

1. Move to an unoccupied lab table that has the double pendulum setup. If one is not immediately available, make sure you and your lab partner have completely filled in Vibrating Strip part of the Hand-in Sheet and answered the questions.
2. Examine the setup and think of the possible ways the system can vibrate. Make sure the motion sensor head is pointed at the reflector (mounted on one of the pendulums) is at the same level as the pendulums, and is aligned with them (all three make an approximately straight line. The sensor should also be no closer than 15 cm to the reflector.
3. Practice setting the pendulums in motion - make sure that the spring is only gently stretched during the oscillation, and never collapses (just hangs limp between the two

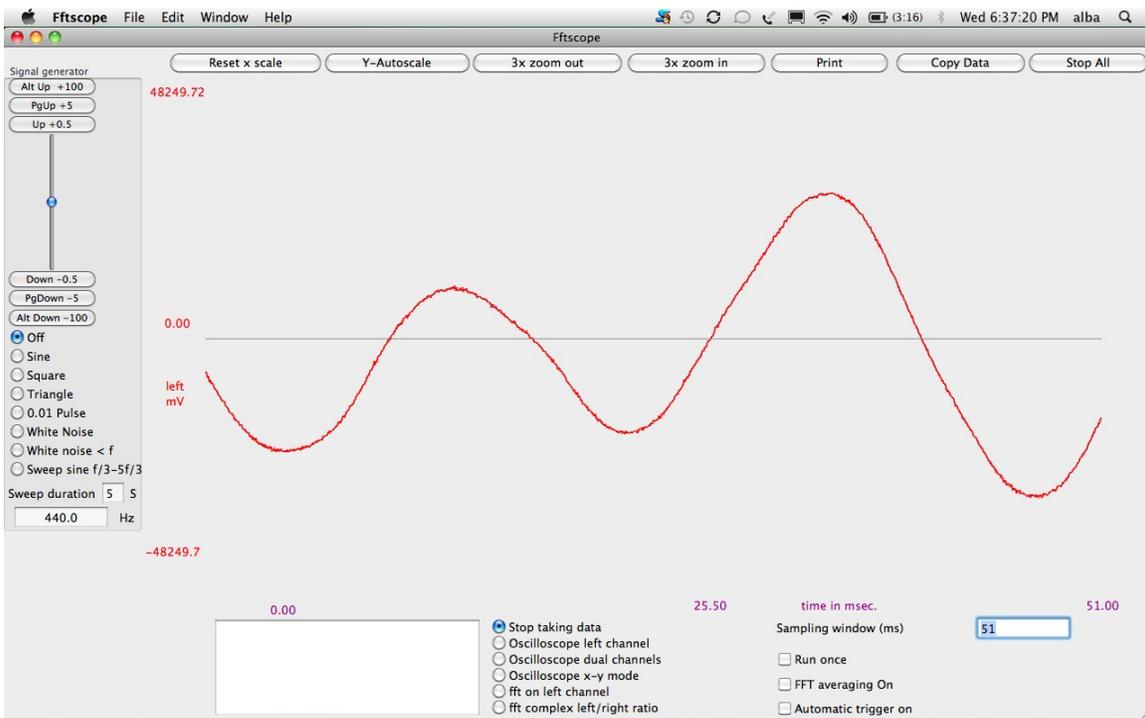
pendulums). **Large amplitude is not necessary, since the motion sensor and FFT are sensitive to small ones.**

4. Open "FFT Template.xmlbl", which is the same folder as this write-up. You will see both an "oscilloscope"-type screen (Amplitude vs. Time) and an FFT screen (Amplitude vs. Frequency). Once you press the Collect button, data will be taken for 30 seconds (although you can later "zoom in" on a region of the Oscilloscope by clicking on the 1 leftmost and rightmost number values of the Time axis).
5. Put the system into motion, exciting the system in the various modes you thought up. Although the motion in general will be mixed, try to limit it only to that mode, e.g., if you want to see the asymmetric mode, carefully move the pendulums apart from each other with equal displacement and release simultaneously. For each mode, note the FFT frequency and observe the Oscilloscope trace.
6. Set the pendulums to rest, then grab one and pull it out slightly. Let go, and take data. You should see an exchange of energy between the two, with different modes coming and going. Take data and note the FFT frequency peaks.
7. Look at the Oscilloscope trace - you should see a "beat" pattern in which one frequency (smaller waves) is "modulated" (shaped) by a lower frequency (larger waves). Measure the periods of these two waves directly (using the readout when you position the cursor over the wave) peaks, and calculate the periods. How do these compare to the FFT frequencies you obtained in Steps 5 and 6?
8. Now vary the *strength* of the coupling by moving the connection point of the spring higher up each of the pendulums (turn the pair of nuts on each pendulum either clockwise or counterclockwise) and making them oscillate again - what does it do to the resulting motion? Look at your oscilloscope and FFT plots to determine the answer.
9. Finally, *de-couple* the two pendulums by removing the spring from both completely. Obviously it will not be possible now for them to exchange energy. Analyze the motion (Oscilloscope, FFT) of the pendulum with the reflector on in it. At what frequency does it oscillate? How does it compare to the FFT peak frequencies you obtained before?

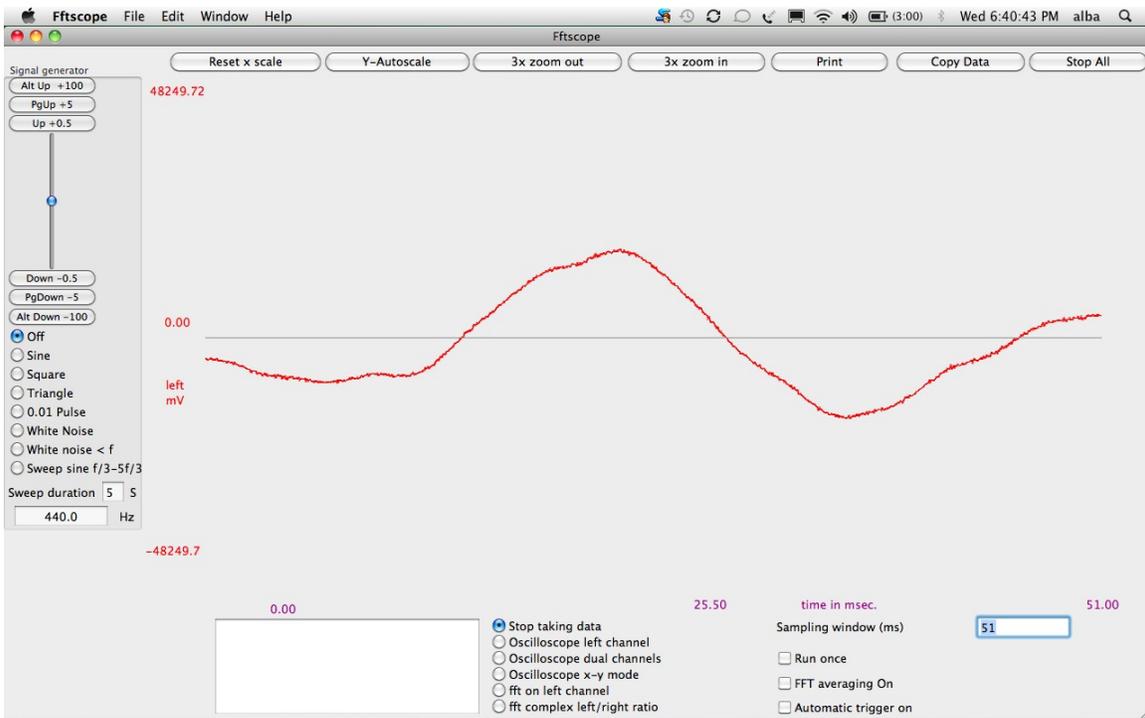
# Sample Data



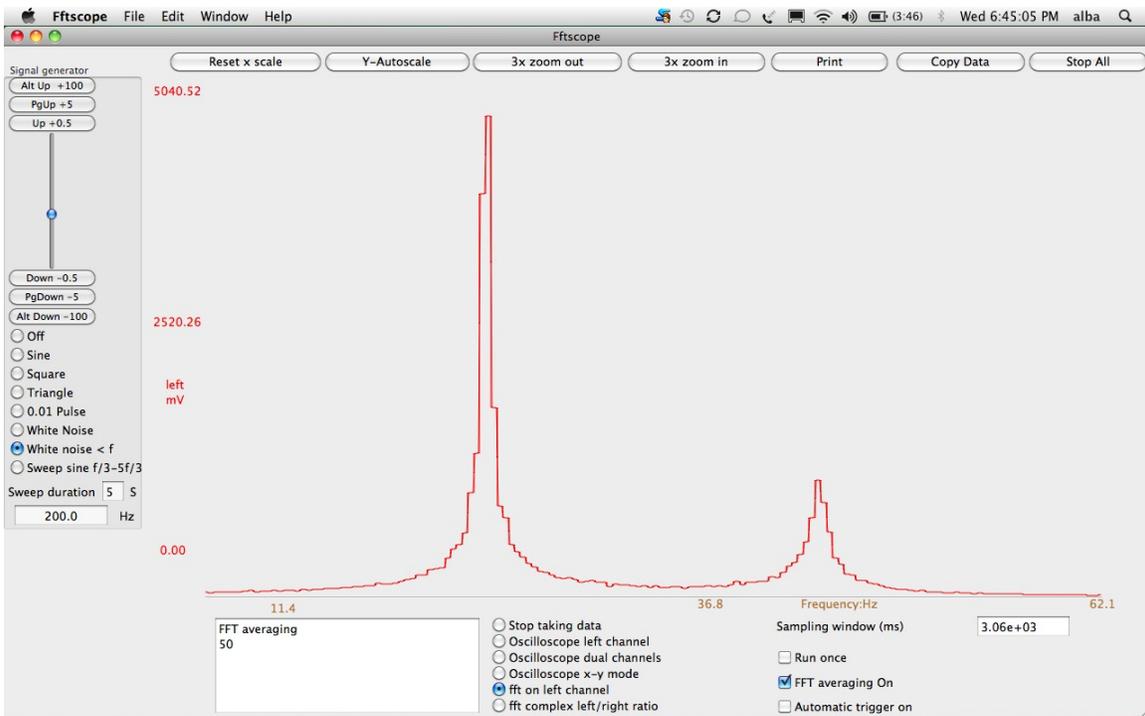
**VIBRATING STRIP:** FftScope software in Oscilloscope mode, showing strip plucked in center to excite symmetric mode.



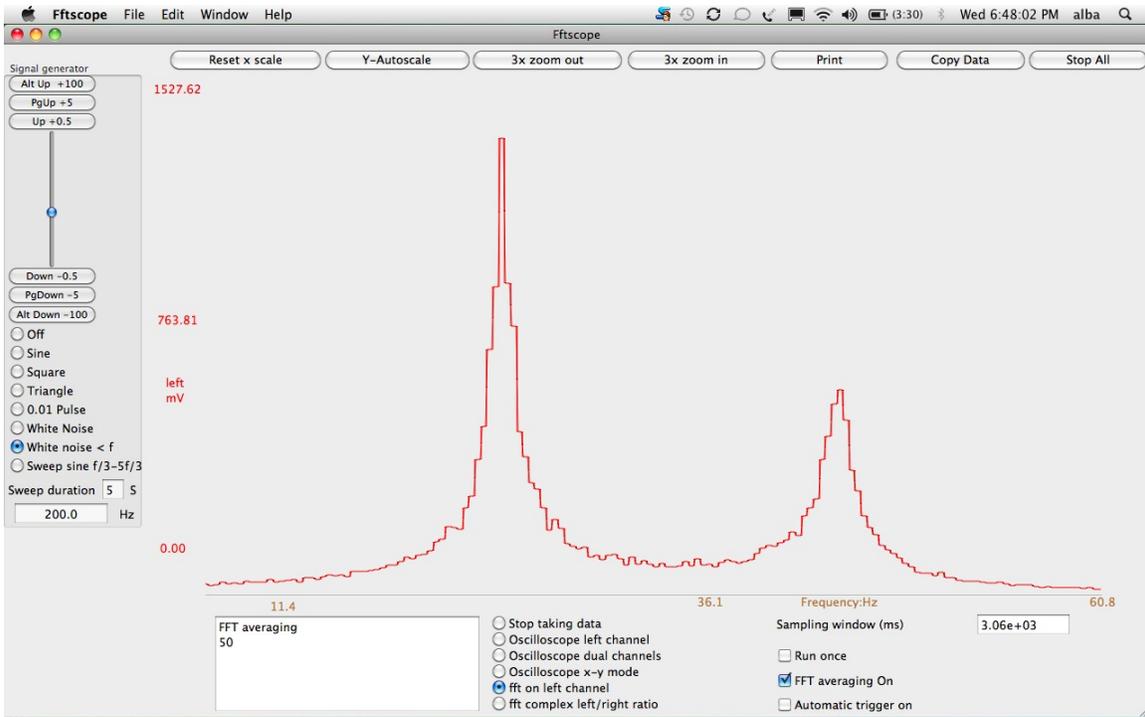
**VIBRATING STRIP:** FftScope software in Oscilloscope mode, showing strip plucked about  $\frac{3}{4}$  of way along its length to excite antisymmetric mode.



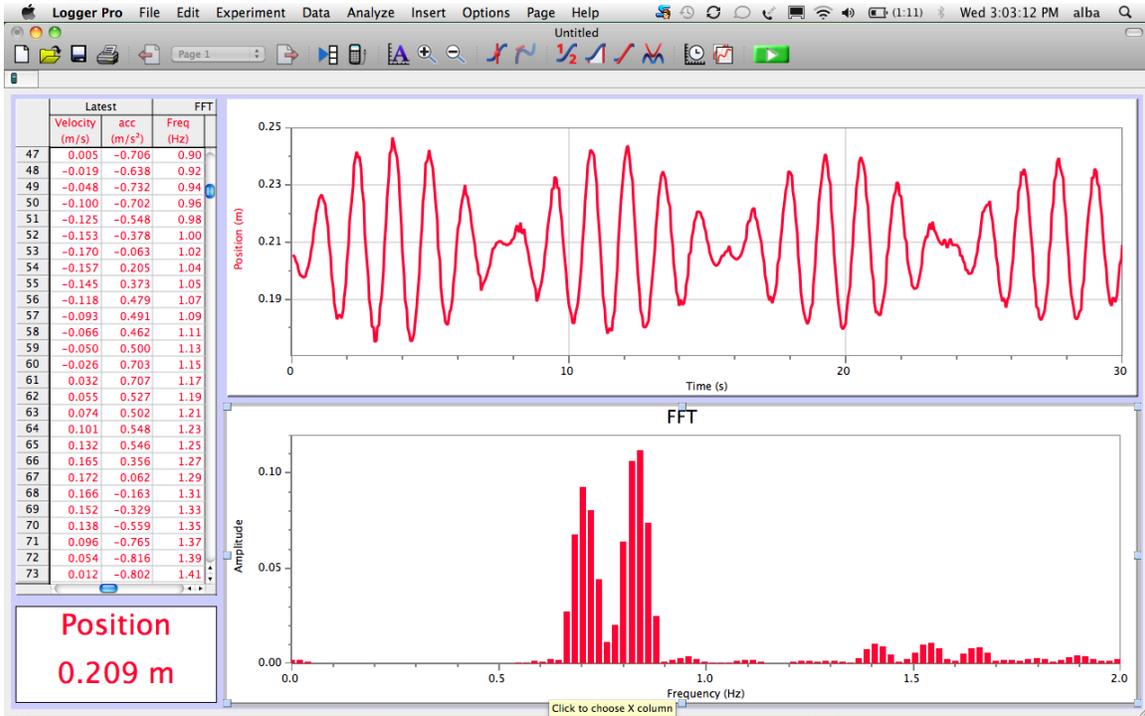
**VIBRATING STRIP:** FftScope software in Oscilloscope mode, showing strip (gently) torsionally twisted near end to excite torsional mode



**VIBRATING STRIP:** FftScope software in FFTmode, showing symmetric and asymmetric mode peaks, response to sinusoidal driving, without damping.



**VIBRATING STRIP: FFTScope software in FFTmode, showing symmetric and asymmetric mode peaks, response to sinusoidal driving, with damping (mechanical and electromagnetic). Note broadening of peaks.**



**COUPLED HARMONIC OSCILLATORS: Logger Pro software showing oscilloscope and FFT traces of mixed modes: symmetric and antisymmetric modes, with harmonics. Note the beats as the individual oscillators exchange energy with each other.**