Zeeman Effect

Equipment:

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Introduction:

In a magnetic field, the spectral lines of atoms split into a number of closely spaced lines. This is called the (anomalous) Zeeman Effect. The Zeeman Effect was very important in the development of Quantum Mechanics, especially Quantum Chemistry. It provided direct evidence that the orbital angular momentum of the atoms (or at least the magnetic moment of atoms) was quantized. The number of lines that a single line split into allowed people to infer the total angular momentum of the energy levels involved in the transition that produced the spectral line.

In this experiment, the student observes the interference pattern from a Fabry-Perot interferometer which results from the 546.1 nm spectral line of a mercury lamp immersed in a uniform magnetic field. The magnetic field is varied from zero to nearly 1 Tesla. Initially, the light is viewed along an axis perpendicular to the magnetic field axis. A polarizer is used to show the three lines due to light that is polarized parallel to the field axis and to show the six lines that are polarized perpendicular to the field axis. The pattern may also be viewed along the field axis where the light is circularly polarized. Finally, the pattern that is polarized perpendicular to the field axis is used to calculate the Bohr magneton. All atomic magnetic moments are integral or half-integral multiples of the Bohr magneton.
Figure 1: Energy Levels for 546.1 nm Hg Spectral Line

Theory:

The 546.1 nm green line in mercury is due to a transition from a \(^3S_1(6s7s)\) energy level to a \(^3P_2(6s6p)\) level as shown. The \((6s7s)\) notation means that mercury has two valence electrons; one in a 6s orbital and one in a 7s orbital. Then orbital angular momentum quantum number for an S state is \(L=0\) and for a P state is \(L=1\). Mercury has two valance electrons which couple here to give a spin quantum number \(S=1\) (called triplet states and annotated with the superscript \(3\)). This results in total angular momentum quantum numbers for the states of \(J=1\) and \(J=2\) respectively. The \(J\) values are indicated by the subscripts. In the presence of a magnetic field, each level splits into \(2J+1\) closely spaced levels. Figure 1 is not to scale. The splitting between the \(2J+1\) levels is much less (~10\(^{-5}\)) than the splitting between the original two levels.

Selection rules allow transitions between states where \(\Delta M_Z = +1, 0, -1\). The transitions where \(\Delta M_Z = 0\) produce photons that are polarized parallel to the magnetic field axis and the lines are more intense. These are the ones we will use for measurement. The \(\Delta M_Z = +1, -1\) transitions produce photons polarized perpendicular to the field axis. Using a polarizer to eliminate the \(\Delta M_Z = +1, -1\) lines makes it easier to do measurements.

Let \(E_S\) be the energy of the un-split (\(B = 0\)) \(^3S_1\) state. This is exactly equal to the \(M_Z = 0\) energy level when the field is not zero. Let \(\Delta E_S\) be the shift in energy from the \(M_Z = 0\) S state when \(B \neq 0\). Similarly for \(E_P\) and \(\Delta E_P\). The energy shift is determined by the component of the magnetic moment of the atom along the magnetic field and is given by:

\[
\Delta E = M_Z g_L \mu_B B
\]

Eq. 1

where \(g_L\) is the Landé g factor and \(\mu_B\) is the Bohr magnetron. The Bohr magnetron is the
elemental unit of magnetic moment (analogous to electron charge e being the elemental unit of electric charge). All atomic magnetic moments are integer or half-integer multiples of $\mu_B$. The Bohr magnetron is the classical magnetic moment that an electron in the first Bohr orbit of hydrogen would possess. Such an electron may be treated as a tiny current loop with current:

$$i = e/T = e/(2\pi r_B/v) = e(\hbar/2\pi r_B m)/(2\pi r_B) = e/h/(m(2\pi r_B)^2)$$

where $T$ is the time for one orbit of the electron, $v$ is the speed of the electron in the first Bohr orbit, $m$ is the electron mass and $r_B$ is the Bohr radius. The magnetic moment of a single current loop is

$$\mu_B = iA = \pi r_B^2 e h/(m(2\pi r_B)^2) = e h/4\pi m = 9.274 \times 10^{-24} \text{ J/T} \quad \text{Eq. 2}$$

where $A$ is the area of the loop. Note that this derivation is not quantum mechanically correct and gives the wrong value for the first Bohr orbit of hydrogen, but the value for the Bohr magnetron is correct.

The Landé $g$ factor is

$$g_L = 1 + \frac{L(L+1) - S(S+1)}{2J(J+1)}$$

For both states the two valence electrons couple to give $S = 1$. $L = 0$ for an $S$ state and $L = 1$ for a $P$ state. The total angular momentum is $J = 1$ for the $S$ state and 2 for the $P$ state. This gives

$$g_S = 2 \quad (\text{for } ^3S_1) \quad g_P = 3/2 \quad (\text{for } ^3P_2)$$

Equation 1 then yields:

$$\Delta E_S = 2B\mu_B \quad \Delta E_P = (3/2)B\mu_B \quad \text{Eq. 3}$$

Where $M_Z = +/- 1$ for the states of interest.

Looking at the $\Delta M_Z = 0$ transitions, the $M_Z = 0$ to $M_Z = 0$ transition will result in the same $\lambda = 546.1 \text{ nm}$ wavelength as the $B = 0$ case. For the $M_Z = +1$ to $M_Z = +1$ transition,

$$h\nu_+ = (E_S + \Delta E_S) - (E_P + \Delta E_P) \quad \text{Eq. 4}$$

For the $M_Z = -1$ to $M_Z = -1$ transition,

$$h\nu_- = (E_S - \Delta E_S) - (E_P - \Delta E_P) \quad \text{Eq. 5}$$

Subtracting the two equations and using Equation 3 yields

$$h(\nu_+ - \nu_-) = 2(\Delta E_S - \Delta E_P) = B\mu_B \quad \text{Eq. 6}$$
Converting to wavelengths

\[ h(c/\lambda_+ - c/\lambda_-) = hc(\lambda_- - \lambda_+)/(\lambda_+)(\lambda_-) = \frac{hc\Delta\lambda/\lambda^2}{\lambda} = B\mu_B \]

\[ \Delta\lambda = \frac{B\mu_B \lambda^2}{hc} \]

Eq. 7

where \((\lambda_-)(\lambda_+) = \lambda^2\) to extremely good approximation.

\[
\Delta l = \frac{DA + AB - CB}{\cos \theta} = d/cos \theta + d/cos \theta - 2d (\sin \theta) \tan \theta = 2d/cos q - 2d \sin^2\theta/cos \theta
\]

\[
\Delta l = 2d \left(1 - \sin^2\theta\right)/\cos \theta = 2d \cos \theta = 2d(1 - \theta^2/2)
\]

Eq. 8

where we have kept only two terms in the Taylor series expansion \(\cos \theta = 1 - \theta^2/2! + \ldots\) since \(\theta\) is very small. The two paths will be in phase and produce an interference maximum if and only if

\[
\Delta l = (n \pm k)\lambda \text{ where } k = 0, 1, 2, 3\ldots \text{ and } n \text{ is a large integer approximately equal to}
\]
2d/\lambda. Equation 8 then yields

\[ 2d(1 - \theta_k^2/2) = (n - k)\lambda \text{ where } k = 0, 1, 2, 3\ldots \quad \text{Eq. 9} \]

where \( \theta_k \) is the angle to the \( k \)th ring in the pattern (see Figure 3, \( k=0 \) for the innermost ring. Here the negative sign must be chosen for the \((n - k)\) term since as \( \theta \) increases, the left of Equation 9 decreases. Thus, the right hand side must decrease as \( k \) increases. Note that the Fabry-Perot interferometer can measure much smaller changes in \( \lambda \) because \( \lambda \) is multiplied by a large \( n \) resulting in a much larger change in \( \theta \). This increases its resolution by roughly \( 2d/\lambda \sim 10^4 \).

Figure 4: Camera Geometry

The camera has a focal length \( f \). Since the angle is small, \( \theta_k = R_k/f \) to very good approximation and Equation 9 becomes

\[ 2d(1 - R_k^2/2f^2) = (n - k)\lambda \text{ where } k = 0, 1, 2, 3\ldots \quad \text{Eq. 10} \]

We do not know the effective focal length \( f \), but can use the \( B = 0 \) pattern shown in Figure 3 to evaluate it. For the \( k = 0 \) ring, Equation 10 is

\[ 2d(1 - R_0^2/2f^2) = n\lambda \quad \text{Eq. 11} \]

Subtracting Equation 10 from Equation 11

\[ d/f^2\lambda = C_0 = k/(R_k^2 - R_0^2) \quad \text{Eq. 12} \]

By measuring the \( R \) values in the \( B = 0 \) pattern, the constant \( C_0 \) can be determined to better than 0.5%.
With an applied magnetic field, through a polarizer parallel to the magnetic field, the spectral lines each split into three lines. The center line in each triplet has the same wavelength as for the B = 0 case. The inner ring with radius $R_k$ has a slightly longer wavelength and the outer ring ($R_{k+}$) has a slightly shorter wavelength. Equation 10 is still valid and yields

$$2d(1 - R_k^2/2f^2) = (n - k)\lambda_\text{-} \text{where} k = 0, 1, 2, 3\ldots$$

and

$$2d(1 - R_{k+}^2/2f^2) = (n - k)\lambda_\text{+} \text{where} k = 0, 1, 2, 3\ldots$$

Subtraction yields

$$(d/f^2)(R_{k+}^2 - R_k^2) = (n - k)(\lambda_\text{-} - \lambda_\text{+}) \text{where} k = 0, 1, 2, 3\ldots$$

Using Equation 7 and the excellent approximation $(n - k)\lambda_\text{=} n\lambda = 2d$

$$(d/f^2)(R_{k+}^2 - R_k^2) = (n - k)(B\mu_B \lambda^2/\hbar c) = (B\mu_B \lambda 2d/\hbar c)$$

and solving for the Bohr magneton

$$\mu_B = \frac{\hbar cC_0}{2Bd} (R_{k+}^2 - R_k^2) \quad \text{Eq. 13}$$

where $C_0$ was determined previously (Equation 12) and $d = 1.995 \text{ mm}$ is given for the Fabry-Perot etalon.
Setup:

![Figure 6: Setup](image)

Caution: Do not stare at the mercury lamp when it is lit. The mercury lamp has bright emission lines in the ultraviolet which can damage your eyes permanently!

1. Place the electromagnet in a convenient location. The locking screw on the base should be away from the direction where the track will be placed (see Figure 7).
   
   **Caution:** The electromagnet weighs 75 pounds!
   
   **Important Note:** Do not lose the Data Report sheet that comes with the electromagnet! You will need it to complete the experiment! Record the serial number of the electromagnet.

2. Use a level to level the electromagnet base in both directions using the screw feet on the electromagnet base. This is only necessary to allow easy observation of the axial field. Attach the spacer block to the track and place slightly curved end of the spacer block against the electromagnet base so the index marks line up.

3. Attach the spacer block to the track and place slightly curved end of the spacer block against the electromagnet base so the index marks line up.

4. Release the locking screw on the base of the electromagnet. With your eye near the end of the track away from the electromagnet and centered on the track, rotate the electromagnet until the gap in the electromagnet is parallel to the track. Lock the locking screw.

5. **Note:** do not touch the glass envelope of the mercury lamp. Oil and moisture from your fingers will degrade the lamp’s performance. Insert the mercury lamp in the holder between the poles of the electromagnet. It should be centered between the pole pieces.

6. Attach the lens/polarizer assembly to the track. The lens should be on the side nearest the electromagnet. See the Appendix for directions on setting up the lens/polarizer if it is not already assembled.
7. Measure the height of the center of the axial iron core plug (silver knob) as shown in Figure 7.

Figure 7: Setting Height

Figure 8: Lens Plane Adjust

8. The optical axis of the lens should go through the middle of the mercury lamp. To do this, first release the two thumbscrews on the base of the lens assembly. Raise the lens assembly so a centimeter or so of the shaft shows (so you can get your fingers on the upper thumbscrew). Rotate the assembly until a book held against the base touches both of the black adjustment screws on the side of the assembly as in Figure 8. Lock the upper thumbscrew. Using the silver ring on the base of the assembly, adjust the height of the assembly until its center (the hole on the near side or the hash mark on the other side) is at the height measured in Step 7. Lock the lower thumbscrew.

9. Adjust the two black adjustment screws on the side of the assembly so the polarizer is roughly parallel to the lens plane by adjusting the gap between the lens holder and the polarizer holder. The polarizer is then roughly parallel to the axis of the electromagnet. Rotate the polarizer so that the 90 degree mark is by the hash mark at the top.

10. Set the lens/polarizer assembly about 30 cm from the magnet end of the track. This lens is used to gather light.

11. Use the large black knob on the bottom of the lens assembly to center the lens over the track by moving the lens left or right.

12. The mercury lamp has wires up two sides. Looking through the lens, rotate the mercury lamp in its holder until the wires are out of the line of sight. It helps to backlight the lamp with a flashlight.

13. Attach the Fabry-Perot interferometer assembly to the track. The three silver knobs should be on the side away from the electromagnet. Position the assembly so the index mark is at 40 cm
on the track scale. See the appendix for directions on setting up the filter/interferometer if it is not already assembled.

14. Adjust the assembly in a manner similar to adjusting the lens assembly. First release the two thumbscrews on the base. Raise the interferometer assembly so it is roughly at the same height as the lens assembly. Rotate the assembly until a book held against the base touches both of the black adjustment screws on the side of the assembly as in Figure 8. Lock the upper thumbscrew. Using the silver ring on the base of the assembly, adjust the height of the assembly until its center (the hole on the near side or the hash mark on the other side) is at the height measured in step 7. Lock the lower thumbscrew. Adjust the two black adjustment screws on the side of the assembly so the interferometer is roughly parallel to the filter by adjusting the gap between the interferometer holder and the filter holder.

15. Plug the white plugs from the mercury lamp into the 1500V outputs on the power supply. It does not matter which is which. Plug the red and black leads from the corresponding plugs on the electromagnet to the red and black plugs on the power supply. Turn on the power supply. Caution: Due to heat generated in the electromagnet, do not run the power supply except when you are taking measurements. In general, minimize the time that the power supply is operating at a current of more than 3 A since a thermal breaker will kick off and you will have to wait for it to reset before proceeding with the experiment. For now the power supply should be set for 0.00 A. The lamp may require a few minutes to reach full brightness. **Do not look directly at the lit mercury lamp as it emits ultraviolet.**

16. Look through the interferometer (perfectly safe because only the 546.1 nm light gets through the interference filter…no ultraviolet). You should see a green bulls-eye (like Figure 3).

17. Attach the camera (with lens) to the track. Adjust it so its center is at the same height as the other two assemblies. Align it so it is parallel to the track. Position it at the end of the track (near the 57 cm mark).

18. Connect the camera to the computer via the USB port. Start the PASCO Capstone application and create a movie display. Select Record Movie with Synced Data. The image from the camera should look like the one in Figure 9. In Capstone, click on the gear icon at the top of the movie panel to open the properties, click Movie Recording, and set Frame Size to 1280 x1024. Use the large black knob to move the lens left and right to make the pattern equally bright on each side. Use the other adjustment screws on the lens to make the illumination even vertically and horizontally.
19. In Capstone, create a table (Table I) and create a User-Entered Data Set called \( k \) (with no units). Create another User-Entered Data Set in the second column called Radius (with units of m).

20. In Capstone, create a calculation:
\[
C = \frac{[k]}{([\text{Radius} \ (m)]^2 - [R_o \ (m)]^2)} \quad \text{with units of} \ 1/m^2
\]
\( R_o=1 \)

*Note the value of \( R_o \) is arbitrarily set to 1 and the actual value will be measured and entered into the calculator later.*

21. Create a third column in Table I and select the calculation \( C \). Now create a new set called “B=0” by clicking on the top of the first column and choosing “Create a new user-entered data set”. Turn on the statistics for the Mean and the Standard Deviation.

22. Create another table (Table II) and create a Run-Tracked User-Entered Data Set called \( R_+ \) (with units of m). Create another Run-Tracked User-Entered Data Set in the second column called \( R_- \) (with units of m).

23. In Capstone, create a calculation:
\[
\mu \text{ Bohr} = \frac{C_2 \ (N)}{([R_+ \ (m)]^2 - [R_- \ (m)]^2)/(2*[d \ (m)]*[B \ (T)])} \quad \text{with units of} \ J/T
\]
\[
C_2 = \frac{[C_0 \ (1/m^2)]*[\text{Planck's constant} \ (J \cdot s)*[\text{Speed of light} \ (m/s)]}{2} \quad \text{with units of} \ 1/m^2
\]
\( C_0 = 1 \)
\( d= 0.00200 \) \quad \text{with units of m}
\( B= 1 \) \quad \text{with units of T}

24. Create a third column in Table II and select the calculation \( \mu \text{ Bohr} \). Now create a new set called “k=1” by clicking on the top of the first column and choosing “Create a new user-entered data set”. Turn on the statistics for the Mean and the Standard Deviation.
Procedure:

1. Dim the room lights.

2. Adjust the aperture on the camera (front ring) to get an intensity bright enough to see, but not burning out. Adjust the back ring on the camera to focus the image.

3. Center the pattern horizontally approximately by moving the camera. To do this, hold the camera, release the upper screw on the base and rotate the camera left or right. Tighten the screw. Then use the black knobs on the interferometer assembly to adjust the vertical position and fine tune the horizontal position.

4. If the pattern is not uniformly illuminated (as in Figure 11), use the large black knob on the lens assembly to adjust the horizontal position of the lens. Use the other adjustment screws on the lens to make the illumination even vertically and horizontally.

5. Increase the power supply current to 5.0 A. The rings should split into triplets as shown in Figure 10. The rings should be sharply defined all the way around. If the pattern is like Figure 11, the plates of the interferometer are not sufficiently parallel. This is adjusted using the three silver knobs on the interferometer. You may also need to adjust the camera aperture and focus. This adjustment is a bit tricky. It is a good idea to put a pencil mark on each of the silver knobs to mark the initial vertical position and keep track of how far you have turned them clockwise or counter-clockwise. That way you can always start over. Caution: if you go too far the knob will pop off. This adjustment is critical for obtaining good results!

6. Reduce the power supply output to 0.00 A.

7. Click Record. The polarizer should still be set to 90°. After a few seconds, increase the power supply output to 5.00 A. After about 2 seconds, click Stop. Return the current to zero. This is Run#1. Open the Data Summary and rename this run "90 Deg".

8. Rotate the polarizer to 0°. Click Record. Wait a few seconds and then increase the power supply to 5.00 A. After about 2 seconds, click Stop. Return the current to zero. This is Run#2. Open the Data Summary and rename this run "0 Deg".

9. Remove the polarizer. Click Record. Wait a few seconds and then increase the power supply to 5.00 A. After about 2 seconds, click Stop. Return the current to zero. This is Run#3. Open the Data Summary and rename this run "No Pol". Replace the polarizer and set it at 90 degrees.

10. Remove the axial iron core plug (silver knob) from the electromagnet. It just slides out, but is a tight fit so it is probably easier to rotate it as you pull out. Rotate the electromagnet by 90° on its base so the axial hole is toward the camera.
11. Click the Preview button in the movie toolbar. If necessary, adjust the position of the electromagnet and/or mercury lamp to give a bright image. You will probably need to open the aperture on the camera.

12. With the polarizer at 90 degrees, click Record and turn up the current to 5.00 A. Then rotate the polarizer to zero and then remove the polarizer. Stop recording and return the current to zero. Rename this run "Magnet Axis".

13. Replace the core plug.

14. Turn the power supply off. **Do not disconnect the camera from the computer!**

Analysis:

1. Click the “Enter Video Analysis Mode” button at the left of the movie toolbar. Select the “90 Deg” run at the bottom. Set the playback slider to the beginning (B = 0 at that point).

2. Click on the Calibration button to turn off the Calibration. This means that the radius tool will not be calibrated. This will affect the value of $C_0$, but not the final result since examination of Equations 12 & 13 shows that the radii are squared in both the denominator and the numerator, so any scale factor cancels out.

3. Click on the Create Measurement Tool button and select Radius Tool. A circle with three handles will appear. There is also a small x marking the center of the circle and a value for the radius of the circle.

4. Click the gear icon (properties) at the right on the video toolbar. Click Overlay. Turn on “Use Magnifier with Tools”. Now click on Radius Tool and set the “Number of Decimal Places” to 3. Click OK to close.

5. Drag the three handles on the Radius Tool until they are all on the k=4 circle (recall that the center circle is k=0) and about 120 degrees apart. Note that when you click on one of the three handles, a magnified image of the video image appears with a small dot in the center of the magnified image marking the position of the handle. The amount of magnification may be varied between 1.5x and 4x using the thumbwheel on the mouse. This allows a finer adjustment of the handle position. The best amount of magnification is determined by the quality of the video image, but should be at least 2x.

6. It is OK if a portion of the k=4 circle is off the screen as long as the three handles are all about 120 degrees apart. If k=4 is not clear, start with k=3. Record the value for R to three decimal places. Move the axes so the origin is at the point marking the center of the circle. All the circles should have the same origin.

7. Using the same circle, repeat for the k=3, k=2, k=1, k=0 circles. Don't define a new Radius Tool since they will be on top off each other. Try to keep the circle origin at the axes origin.
8. Slide the playback slider all the way to the right to show the $B \neq 0$ data for which each line has become a triplet and $B$ is at its max value. Note that the center line in the $k=0$ triplet is still at the position marked by the Radius Tool for the $B=0$ case. What does this imply?

9. Use the Radius Tool to measure the inner and outer circles of the $k=1$ triplets and record the $k=1$ data in Table II. Repeat the measurements two more times to help estimate the uncertainty.

10. In Table II, create a new data set called $k=2$ and use the radius tool to measure the inner and outer circles of the $k=2$ triplets as you did for the $k=1$ triplets in Step 9.

Data:

1. Click open the calculator at the left of the screen. For the magnetic field, enter the value (replace the 1 T) of the magnetic field strength for your electromagnet at the current you used (5.00 A). This value is given on the data sheet that came with the electromagnet. If there is more than one electromagnet in the lab, note that there is a serial number on the electromagnet (silver label) and on the data sheet.

2. For $R_0$, replace the 1 m value with your value for radius of the $k=0$ circle with $B=0$.

3. For $C_0$, replace the value 1 with the mean value of $C$ from Table I.

Conclusions:

1. Do the values for the Bohr magneton from $k=1$ and $k=2$ agree with theory ($9.27 \times 10^{-24}$ J/T)? Discuss fully.

2. Was the center line in the triplet the same as the line for the $B=0$ field? What does this show?

3. On the video, select the “0 Deg” run at the bottom of the page. Move the playback slider to the beginning. Using the Create Measurement Tool button, create a radius tool and put it on the $k=1$ ring. Move the slider to the end of the movie. How many energy levels do you see now for the $k=1$ band? Does this agree with the discussion in the theory section? It may help to use the magnifier. The outside bands are more clear with the $k=0$ band, but the $k=0$ inside bands may not show if the $k=0$ band is too close to the center.

4. Now choose "No Pol" run where the polarizer was removed. Can you see all the bands? How many are there? Does it match theory?

5. Now select "Magnet Axis" at the bottom showing the axial field. Start with the slider at the beginning and move it to the end. Recall that you rotated the polarizer through 90° as you took the movie. Do you see any change in the pattern? What does this tell you?
Appendix: Assembling the System Components

Lens/Polarizer Assembly

Put the collimating lens and the polarizing filter into the precision kinematic optical mount that has the horizontal translator (as shown in Figure 12). The mount is adjustable in a horizontal direction using the large knob at the bottom. Lock the set screw that holds the lens in place, but leave the polarizer set screw loose so the polarizer may be rotated. Complete the Lens/Polarizer Assembly as shown in Figure 13.

Fabry-Perot Interferometer Assembly

Screw the green interference filter onto the Fabry-Perot interferometer (Figure 14). Attach the interferometer to the mount as in Figure 15. The silver thumbscrews shown in Figure 16 will be adjusted later. Do not change them now.
Camera Assembly

Attach the lens to the CMOS camera body. Complete the Camera Assembly as shown in Figure 17.