Permittivity	ω_1	Q_1	$\varepsilon'(\omega_1)$	$\varepsilon''(\omega_1)$
Q_0^{-1} dominant	$\dfrac{\omega_0}{\sqrt{\varepsilon'}}$	$\dfrac{Q_0}{(\varepsilon')^{1/4}}$	$\left(\dfrac{\omega_0}{\omega_1}\right)^2$ and $\left(\dfrac{Q_0}{Q_1}\right)^4$	–
ε'' dominant	$\omega_0 \dfrac{\sqrt{\varepsilon'}}{\lvert\varepsilon_\mathrm{r}\rvert}$	$\dfrac{\varepsilon'}{\varepsilon''}$	$\dfrac{\left(Q_1 \dfrac{\omega_0}{\omega_1}\right)^2}{1+Q_1^2}$	$\dfrac{Q_1\left(\dfrac{\omega_0}{\omega_1}\right)^2}{1+Q_1^2}$
general	$\omega_0 \dfrac{\sqrt{\varepsilon'}}{\lvert\varepsilon_\mathrm{r}\rvert}$	$\left[\dfrac{1}{Q_0}\left(\dfrac{\lvert\varepsilon_\mathrm{r}\rvert^2}{\varepsilon'}\right)^{1/4} + \dfrac{\varepsilon''}{\varepsilon'}\right]^{-1}$	$\dfrac{\left(\dfrac{\omega_0}{\omega_1}\right)^2}{1+\left[\dfrac{1}{Q_1}-\dfrac{1}{Q_0}\left(\dfrac{\omega_0}{\omega_1}\right)^{1/2}\right]}$	$\dfrac{\left(\dfrac{\omega_0}{\omega_1}\right)^2\left[\dfrac{1}{Q_1}-\dfrac{1}{Q_0}\left(\dfrac{\omega_0}{\omega_1}\right)^{1/2}\right]}{1+\left[\dfrac{1}{Q_1}-\dfrac{1}{Q_0}\left(\dfrac{\omega_0}{\omega_1}\right)^{1/2}\right]}$
Permeability	ω_2	Q_2	$\mu'(\omega_2)$	$\mu''(\omega_2)$
Q_0^{-1} dominant	$\dfrac{\omega_0}{\sqrt{\mu'}}$	$Q_0\,(\mu')^{3/4}$	$\left(\dfrac{\omega_0}{\omega_2}\right)^2$ and $\left(\dfrac{Q_2}{Q_0}\right)^{4/3}$	–
μ'' dominant	$\dfrac{\omega_0}{\sqrt{\mu'}}$	$\dfrac{\mu'}{\mu''}$	$\left(\dfrac{\omega_0}{\omega_2}\right)^2$	$\dfrac{1}{Q_2}\left(\dfrac{\omega_0}{\omega_2}\right)^2$
general	$\dfrac{\omega_0}{\sqrt{\mu'}}$	$\left[\dfrac{1}{Q_0}\dfrac{1}{(\mu')^{3/4}} + \dfrac{\mu''}{\mu'}\right]^{-1}$	$\left(\dfrac{\omega_0}{\omega_2}\right)^2$	$\left(\dfrac{\omega_0}{\omega_2}\right)^2\left[\dfrac{1}{Q_2}-\dfrac{1}{Q_0}\left(\dfrac{\omega_2}{\omega_0}\right)^{3/2}\right]$

TABLE I. Top: Summary of the relationships between ω_0, Q_0, ω_1, Q_1 and the complex relative permittivity. Bottom: Summary of the relationships between ω_0, Q_0, ω_2, Q_2 and the complex relative permeability.

magnetic particles present to determine ω_0 and Q_0. Next, the resonance would be remeasured with the magnetic particles suspended in the liquid to find ω_2 and Q_2. Finally, μ' and μ'' could then be determined using the appropriate relationships given in Table I.

V. SUMMARY

Methods for determining the complex relative permittivities and permeabilities of materials using a SRR have been described. In general, the SRR signal response is non-Lorentzian. However, in a wide variety of cases the signal is very accurately approximated by a Lorentzian. For dielectric materials, the limiting cases of $Q_0^{-1} \gg \varepsilon''/\varepsilon'$ and $Q_0^{-1} \ll \varepsilon''/\varepsilon'$ as well as the general case $Q_0^{-1} \sim \varepsilon''/\varepsilon'$ were all considered. Teflon, 2-propanol, and water were used as examples of these three cases and it was shown how measurements of the in-air and in-liquid resonant frequencies and quality factors can be used to experimentally determine ε' and ε''. An equivalent analysis for the case of a SRR submerged in a ferromagnetic suspension was also presented.

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[1] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184–4187 (2000).

[2] D. A. Bonn, D. C. Morgan, and W. N. Hardy, Rev. Sci. Instrum. **62**, 1819–1823 (1991).

[3] W. N. Hardy, D. A. Bonn, D. C. Morgan, R. Liang, and K. Zhang, Phys. Rev. Lett. **70**, 3999–4002 (1993).

[4] M. Burresi, D. van Oosten, T. Kampfrath, H. Schoenmaker, R. Heideman, A. Leinse, and L. Kuipers, Science **326**, 550–553 (2009).

[5] J. S. Bobowski, Am. J. Phys. **81**, 899–906 (2013).

[6] M. C. Ricci and S. M. Anlage, Appl. Phys. Lett. **88**, 264102 (2006).

[7] W. N. Hardy and L. A. Whitehead, Rev. Sci. Instrum. **52**, 213–216 (1981).

[8] The complex permittivity has been modelled using the function $\varepsilon_\mathrm{r} = \varepsilon_\infty + (\varepsilon_\mathrm{s} - \varepsilon_\infty)/(1 + j\omega\tau)$.

[9] F. Buckley and A. A. Marriott, *Tables of Dielectric Dispersion Data for Pure Liquids and Dilute Solutions* (National Bureau of Standards Circ. No. 589, 1958).

[10] R. Buchner, J. Barthel, and J. Stauber, Chem. Phys. Lett. **306**, 57–63 (1999).